# Investigating magnetic fluctuations in gyrokinetic simulations of tokamak SOL turbulence

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PPPL Theory Research & Review Seminar October 2020



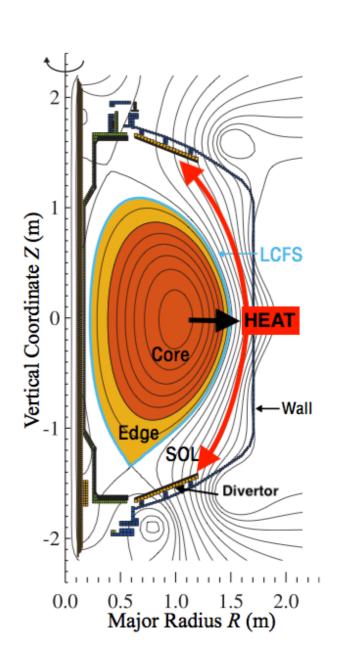






### Why is SOL turbulence important?

- Plasma properties in the tokamak edge/ scrape-off layer (SOL) constrain component lifetime and reactor performance
- Heat exhausted in SOL could damage divertor plates if heat flux width is too narrow
  - Can SOL turbulence broaden the heat flux width?
  - Can electromagnetic effects be important for SOL turbulence?





### Modeling the edge/SOL is challenging

- Gyrokinetic (GK) theory and simulation are important first-principles tools for studying turbulence and transport in fusion plasmas, but most present codes optimized for core, small fluctuations (delta-f)
- Edge/SOL more challenging: large-amplitude fluctuations, open field lines, plasma-wall interactions, X-point geometry, atomic physics, transition from kinetic to fluid regimes → need specialized full-f GK codes
- Including electromagnetic effects (allowing magnetic field to fluctuate) also challenging → all GK SOL results to-date have been electrostatic (no magnetic fluctuations)
- Several GK codes making great progress in edge/SOL
  - XGC1, COGENT, ELMFIRE, GENE, Gkeyll, etc.
- Essential to have several independent codes to attack from different perspective and cross-check on difficult turbulence problems!

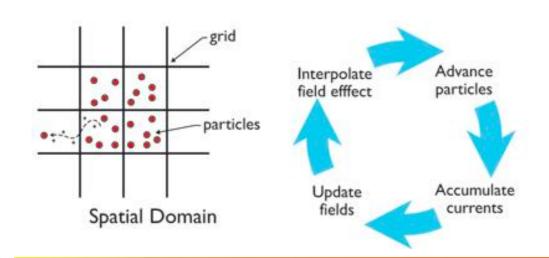
Particle-in-cell (Lagrangian)

Continuum (Eulerian)



Particle-in-cell (Lagrangian)

- Sample phase space with ensemble of  $N_p$  'superparticles'
- Fields on 3D grid, particles move through grid
- Historically, EM fluctuations challenging in GK PIC codes due to numerical "Ampere cancellation" problem
  - Codes like ORB5 and XGC1 have made good recent progress to mitigate issue

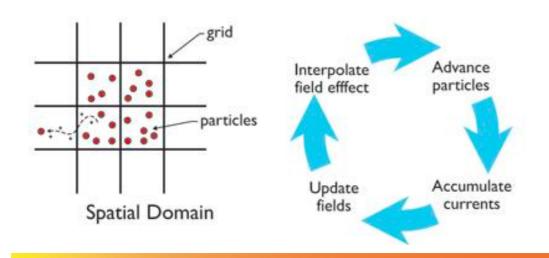


Continuum (Eulerian)



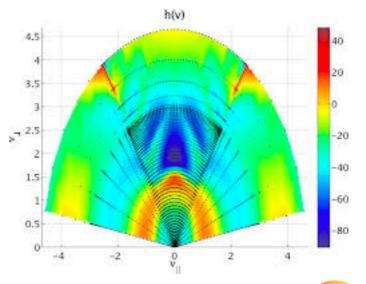
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#### Continuum (Eulerian)

- Discretize distribution function  $f(x,y,z,v_{||},\mu)$  on 5D phase space grid
- Can solve with standard PDE methods, e.g. spectral, finite volume, discontinuous Galerkin, etc.
- Continuum electromagnetic GK codes have mostly avoided the Ampere cancellation problem



Barnes et al, 2010





https://github.com/ammarhakim/gkyl/

https://gkeyll.readthedocs.io

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- First successful <u>continuum</u> GK code on open field lines
- First electromagnetic GK on open field lines

### Full-f electromagnetic gyrokinetics

EMGK equation,  $f_s = f_s(\mathbf{R}, v_{\parallel}, \mu; t)$ 

$$\frac{\partial f_s}{\partial t} + \dot{\mathbf{R}} \cdot \nabla f_s + \dot{v}_{\parallel} \frac{\partial f_s}{\partial v_{\parallel}} = C[f_s] + S_s$$

with nonlinear phase-space trajectories

$$\dot{\boldsymbol{R}} = \{\boldsymbol{R}, H_s\} = \frac{\boldsymbol{B_0^*} + \delta \boldsymbol{B_\perp}}{B_\parallel^*} v_\parallel + \frac{\hat{\mathbf{b}}}{q_s B_\parallel^*} \times (\mu \nabla B + q_s \nabla \phi)$$

$$\dot{v}_\parallel = \{v_\parallel, H_s\} - \frac{q_s}{m_s} \frac{\partial A_\parallel}{\partial t} = -\frac{\boldsymbol{B_0^*} + \delta \boldsymbol{B_\perp}}{m_s B_\parallel^*} \cdot (\mu \nabla B + q_s \nabla \phi) - \frac{q_s}{m_s} \frac{\partial A_\parallel}{\partial t}$$

where  $\mathbf{B_0^*} = \mathbf{B_0} + (m_s v_{\parallel}/q_s) \nabla \times \hat{\mathbf{b}}$  and  $\delta \mathbf{B_{\perp}} = \nabla A_{\parallel} \times \hat{\mathbf{b}}$ .

- No assumption of scale separation between background and fluctuations
- Taking long-wavelength (drift-kinetic) limit, neglecting gyroaveraging for now
- Using symplectic  $(v_{\parallel})$  formulation of EMGK, so  $\frac{\partial A_{\parallel}}{\partial t}$  appears explicitly

## Full-f electromagnetic gyrokinetics

Quasineutrality equation (long-wavelength):

$$-\nabla \cdot \sum_{s} \frac{m_s n_{0s}}{B^2} \nabla_{\perp} \phi = \sum_{s} q_s \int d^3 v \ f_s \tag{1}$$

Parallel Ampère equation:

$$-\nabla_{\perp}^2 A_{\parallel} = \mu_0 \sum_s q_s \int d^3 v \ v_{\parallel} f_s \tag{2}$$

Can take  $\frac{\partial}{\partial t}$  to get an exact Ohm's law:

$$-\nabla_{\perp}^{2} \frac{\partial A_{\parallel}}{\partial t} = \mu_{0} \sum_{s} q_{s} \int d^{3}v \ v_{\parallel} \frac{\partial f_{s}}{\partial t}$$
 (3)

Writing GK eq. as

$$\frac{\partial f_s}{\partial t} = \frac{\partial f_s}{\partial t}^* + \frac{q_s}{m_s} \frac{\partial A_{\parallel}}{\partial t} \frac{\partial f_s}{\partial v_{\parallel}},\tag{4}$$

where  $\frac{\partial f_s}{\partial t}^*$  denotes all the terms in the gyrokinetic equation except the  $\frac{\partial A_{\parallel}}{\partial t}$  term, can write Ohm's law as

$$\left(-\nabla_{\perp}^{2} + \sum_{s} \frac{\mu_{0} q_{s}^{2}}{m_{s}} \int d^{3}v \ f_{s}\right) \frac{\partial A_{\parallel}}{\partial t} = \mu_{0} \sum_{s} q_{s} \int d^{3}v \ v_{\parallel} \frac{\partial f_{s}}{\partial t}^{\star} \tag{5}$$



#### Ampère cancellation problem

• In  $p_{\parallel}$  formulation, Ampère's law:

$$\left(-\nabla_{\perp}^{2} + \frac{C_{n}}{C_{n}}\sum_{s}\frac{\mu_{0}q_{s}}{m_{s}}\int d^{3}p f\right)A_{\parallel} = \frac{C_{j}}{m_{s}}\mu_{0}\sum_{s}\frac{q_{s}}{m_{s}^{2}}\int d^{3}p p_{\parallel}f$$

- "Cancellation problem" arises when there are small errors in the calculation of the integrals, represented by  $C_n$  and  $C_j$  (which should be exactly 1 in the exact system)
- Recall  $v_{\parallel}$  formulation Ohm's law... same problem...

$$\left(-\nabla_{\perp}^{2} + \frac{C_{n}}{m_{s}}\sum_{s} \frac{\mu_{0}q_{s}^{2}}{m_{s}} \int d^{3}v \ f_{s}\right) \frac{\partial A_{\parallel}}{\partial t} = \frac{C_{j}}{m_{s}} \mu_{0} \sum_{s} q_{s} \int d^{3}v \ v_{\parallel} \frac{\partial f_{s}}{\partial t}^{\star}$$

• The simplest Alfvén wave dispersion relation (slab geometry, uniform Maxwellian background with stationary ions) becomes (with  $\hat{\beta} \equiv \frac{\beta_e}{2} \frac{m_i}{m_e}$ )

$$\omega^{2} = \frac{k_{\parallel}^{2} v_{A}^{2}}{C_{n} + k_{\perp}^{2} \rho_{s}^{2} / \hat{\beta}} \left[ 1 + (C_{n} - C_{j}) \frac{\hat{\beta}}{k_{\perp}^{2} \rho_{s}^{2}} \right]$$

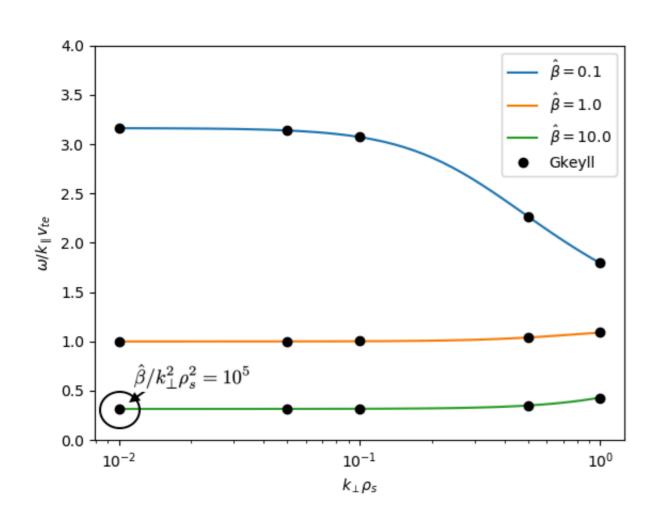
• This reduces to the correct result if integrals calculated consistently, so that  $C_n = C_j$ , but if not there will be errors  $\sim \omega_H$  for modes with  $\hat{\beta}/k_{\perp}^2 \rho_s^2 \gg 1$ .

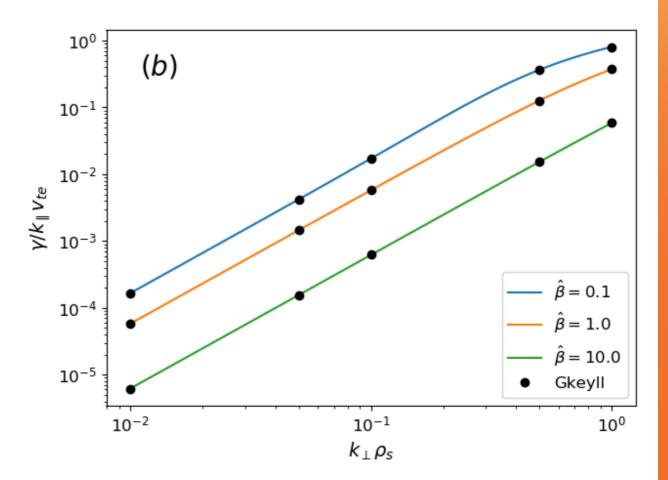
Gkeyll's DG scheme computes integrals consistently so that errors cancel exactly (appendix of Mandell et al, JPP 2020 shows numerical dispersion relation calculation)



#### Linear benchmark: kinetic Alfvén wave

$$\omega^{2} = \frac{k_{\parallel}^{2} v_{A}^{2}}{C_{n} + k_{\perp}^{2} \rho_{s}^{2} / \hat{\beta}} \left[ 1 + (C_{n} - C_{j}) \frac{\hat{\beta}}{k_{\perp}^{2} \rho_{s}^{2}} \right] \qquad (k_{\perp} \rho_{s} \ll 1)$$

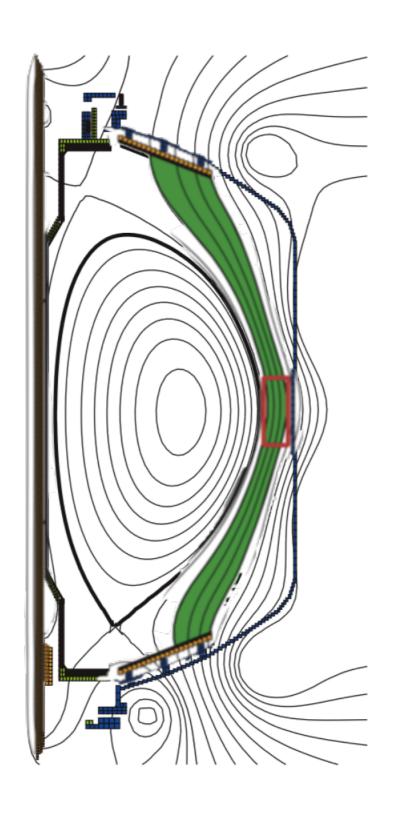




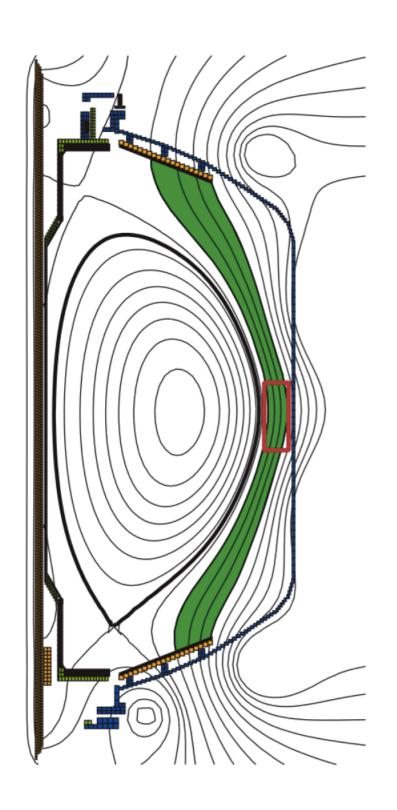
**Exercise 3.1** results match theory very well, even for case with  $\frac{\hat{\beta}}{k_{\perp}^2 \rho_s^2} = \frac{\beta_e/2}{k_{\perp}^2 \rho_s^2} \frac{m_i}{m_e} = 10^5$ 

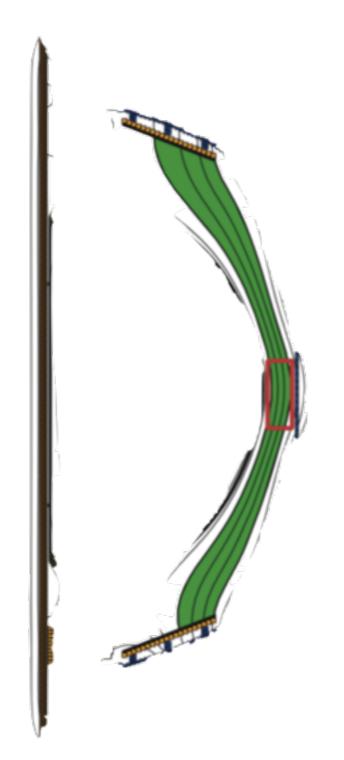
No cancellation problem!





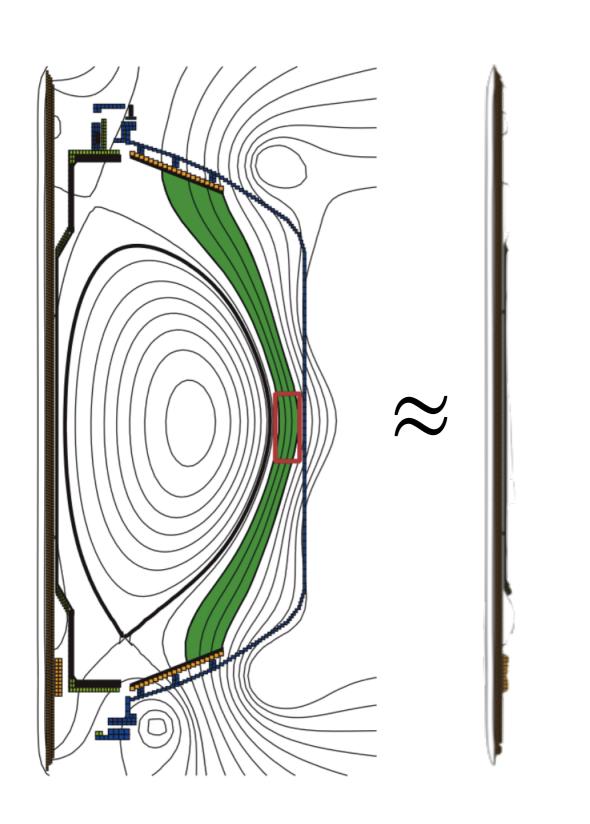


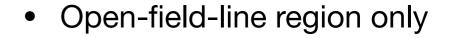




Open-field-line region only

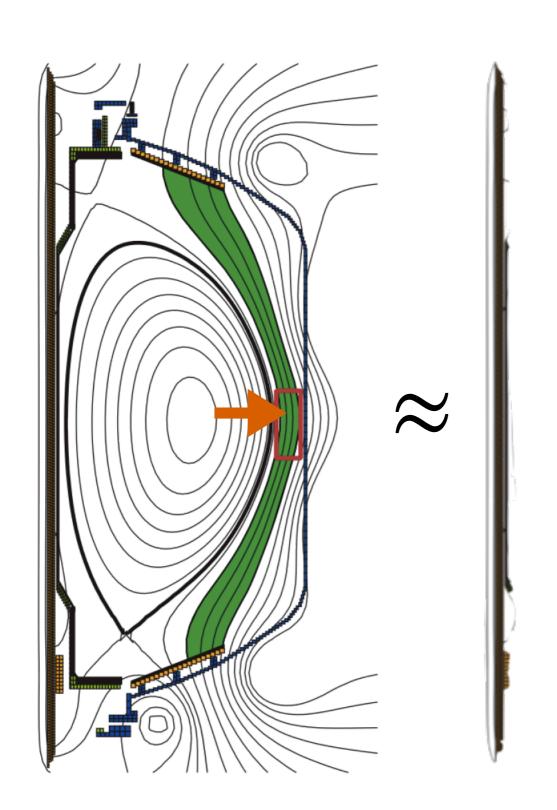


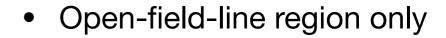




 Simplified helical geometry with vertical flux surfaces, const curvature and no shear (no X point)

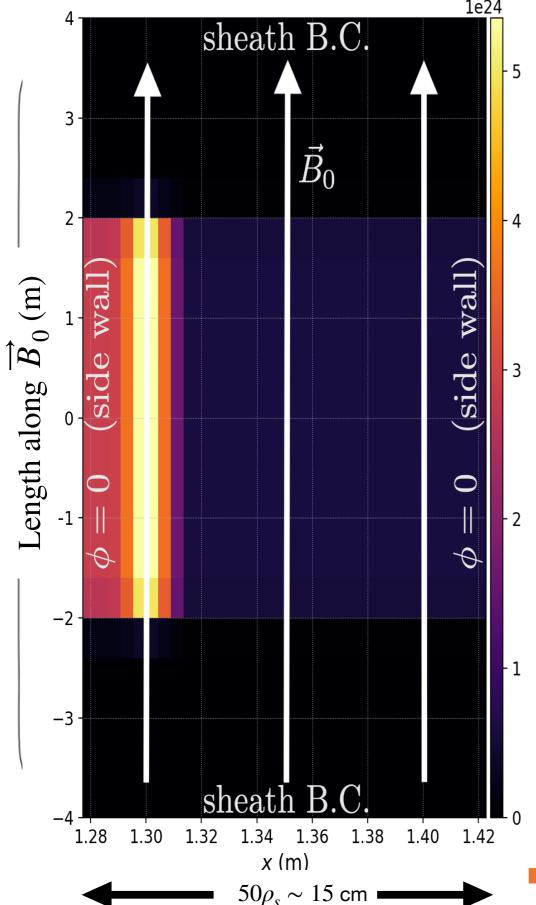






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- Model flux of heat and particles across separatrix with source

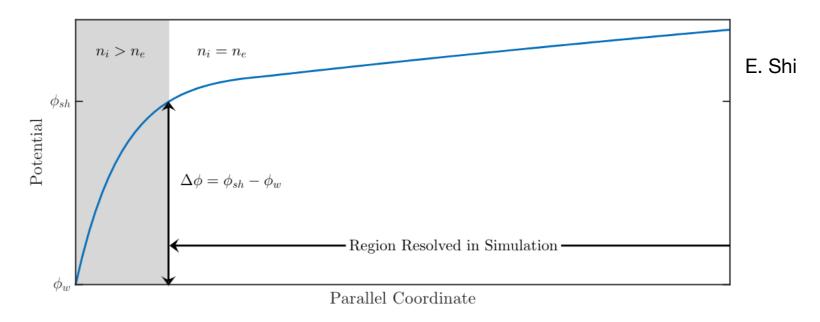




- Open-field-line region only
- Simplified helical geometry with vertical flux surfaces, const curvature and no shear (no X point)
- Model flux of heat and particles across separatrix with source
- Boundary conditions:
  - perfectly conducting walls  $(\phi = A_{||} = 0) \text{ in radial}$  direction, x
  - periodic in binormal direction, y
  - conducting sheath model
     BC along field line, z



### Conducting-sheath boundary condition

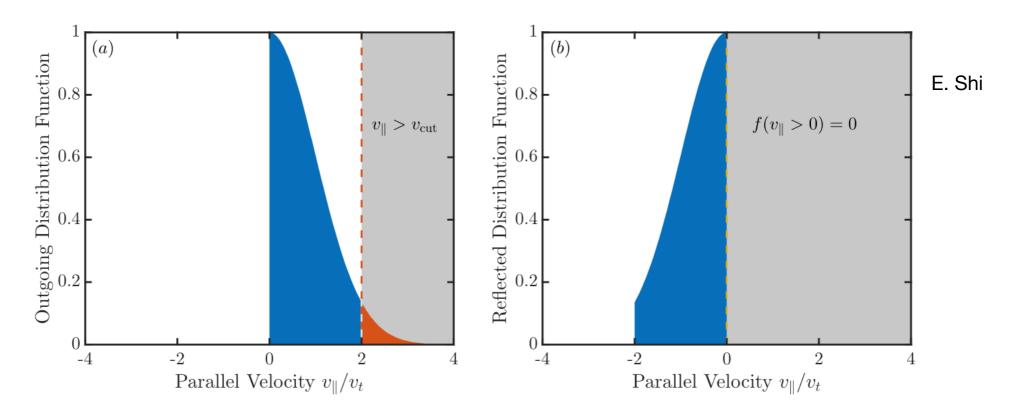


- Need to model non-neutral sheath using BCs (GK assumes quasi-neutrality, cannot resolve sheath)
- Sheath potential should reflect low energy electrons
- Solve Poisson equation on z boundary to get  $\phi_{sh}(x,y) \doteq \phi(z=z_{sh})$ , then use  $\Delta \phi = \phi_{sh} \phi_w$  to reflect electrons with  $mv_{\parallel}^2/2 < |e| \Delta \phi$

$$-\nabla_{\perp} \cdot \sum_{s} \frac{m_s n_{0s}}{B^2} \nabla_{\perp} \phi(z = z_{sh}) = \sum_{s} q_s \int d^3 v \ f_s(z = z_{sh})$$

 Potential self-consistently relaxes to ambipolar-parallel-outflow state, and allows local currents in and out of wall (unlike "logical" sheath model)

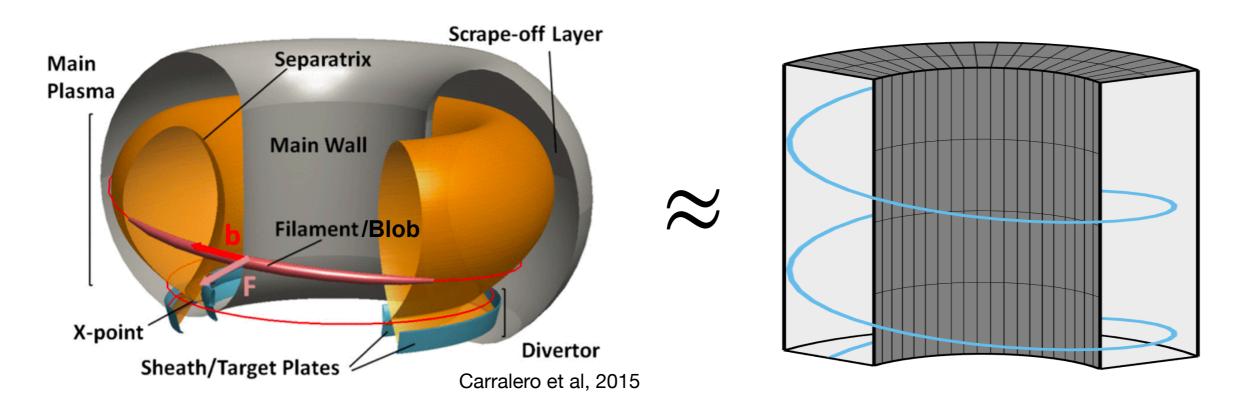
#### Sheath boundary condition for electrons



- (a) Outgoing electrons with  $v_{\parallel}>v_{cut}=\sqrt{2e\Delta\phi/m}$  are lost into the wall
- (b) Rest of outgoing electrons  $0 < v_{\parallel} < v_{cut}$  are reflected back into plasma

Ions: Assuming positive sheath potential (relative to wall), all ions are lost

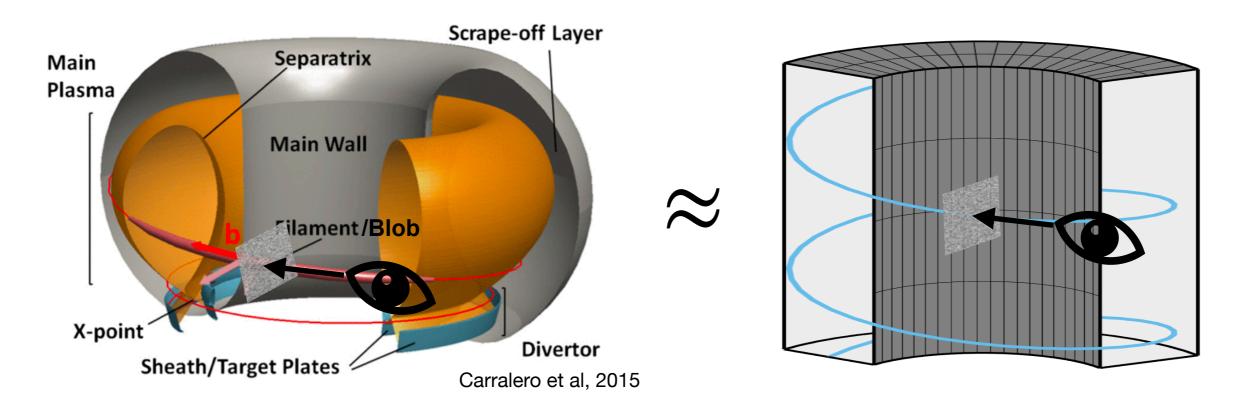




#### Simple helical model of NSTX SOL

- Field-aligned simulation domain that follows field lines from bottom divertor plate, around the torus, to the top divertor plate
- Length along field line ~ connection length  $L_{\parallel}=8$  m (constant, no shear for now)
- All bad curvature → interchange instability, blob dynamics
- Real deuterium mass ratio, Lenard-Bernstein collisions





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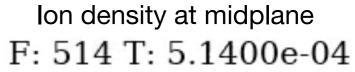


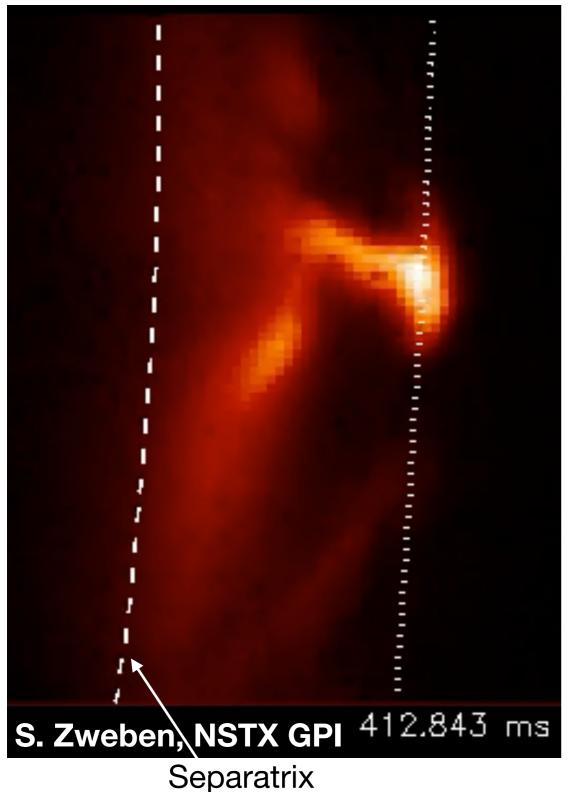
### **NSTX GPI**

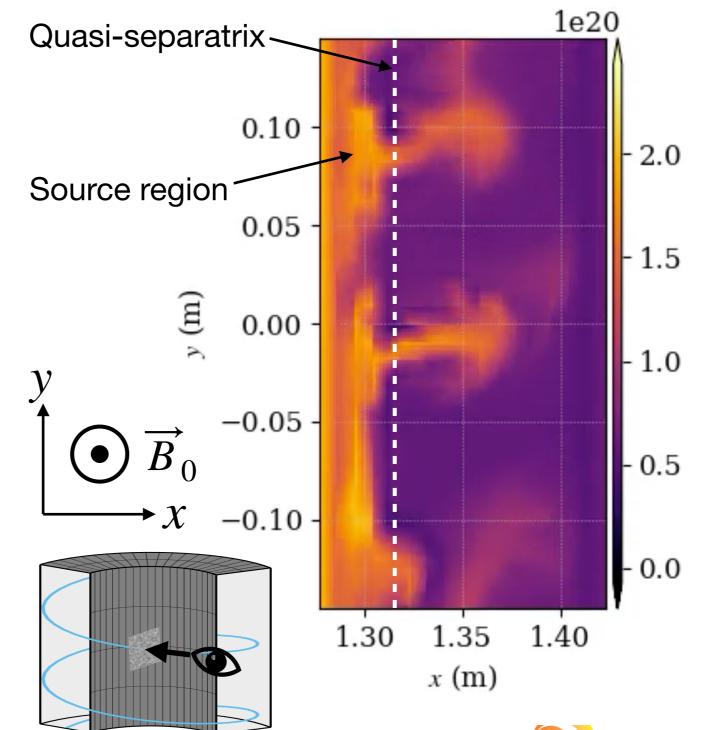
VS



 $\mathrm{D}\alpha$  signal  $\sim$  density at midplane





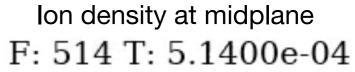


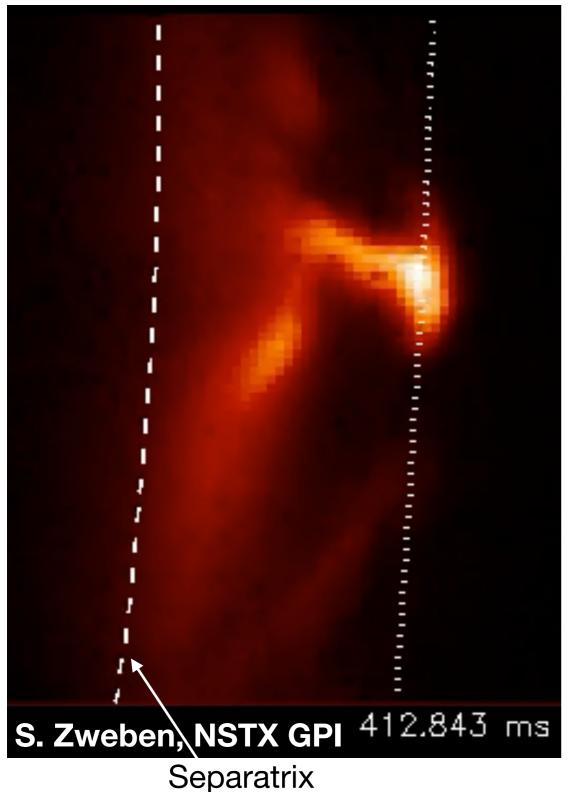
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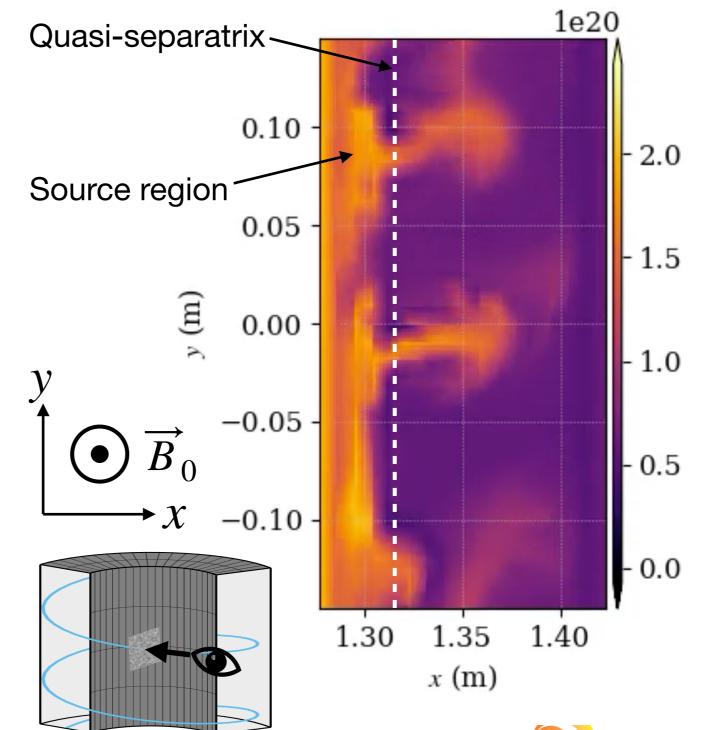
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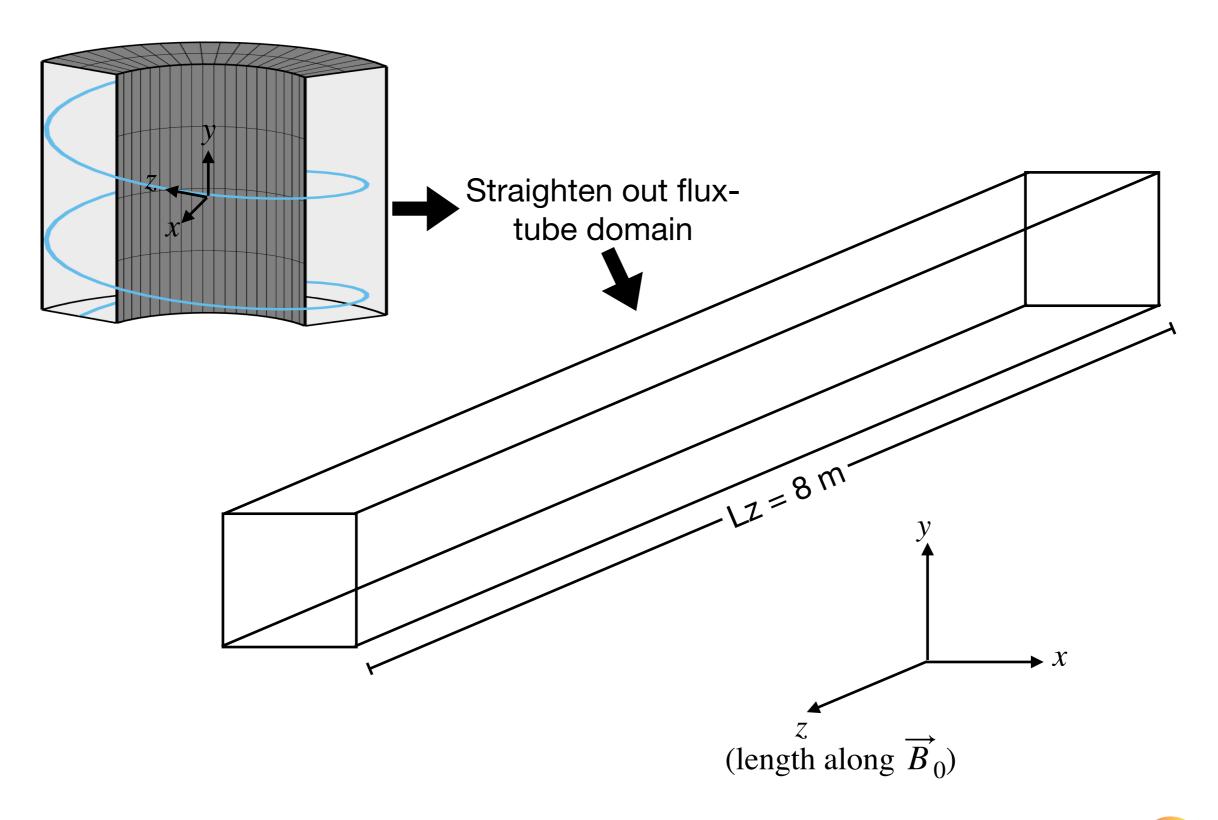


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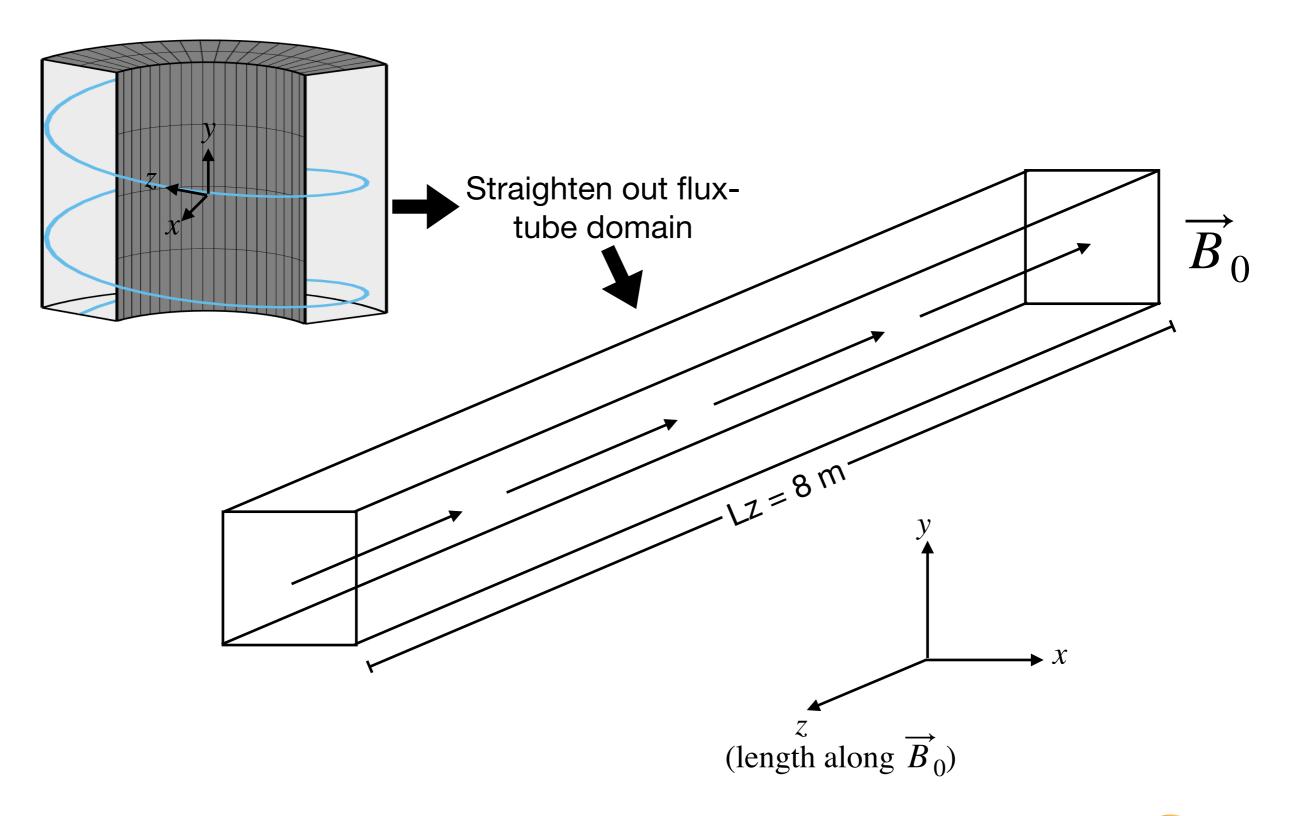




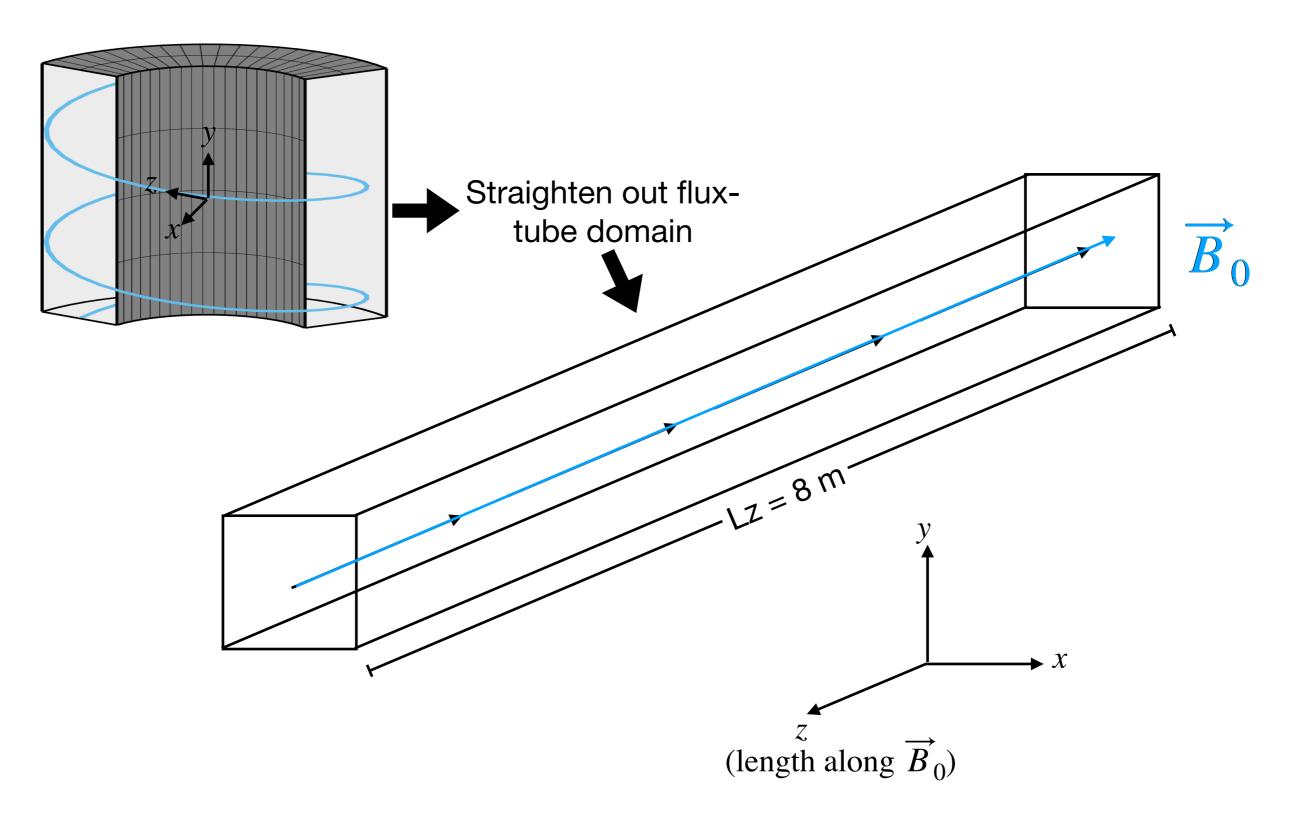




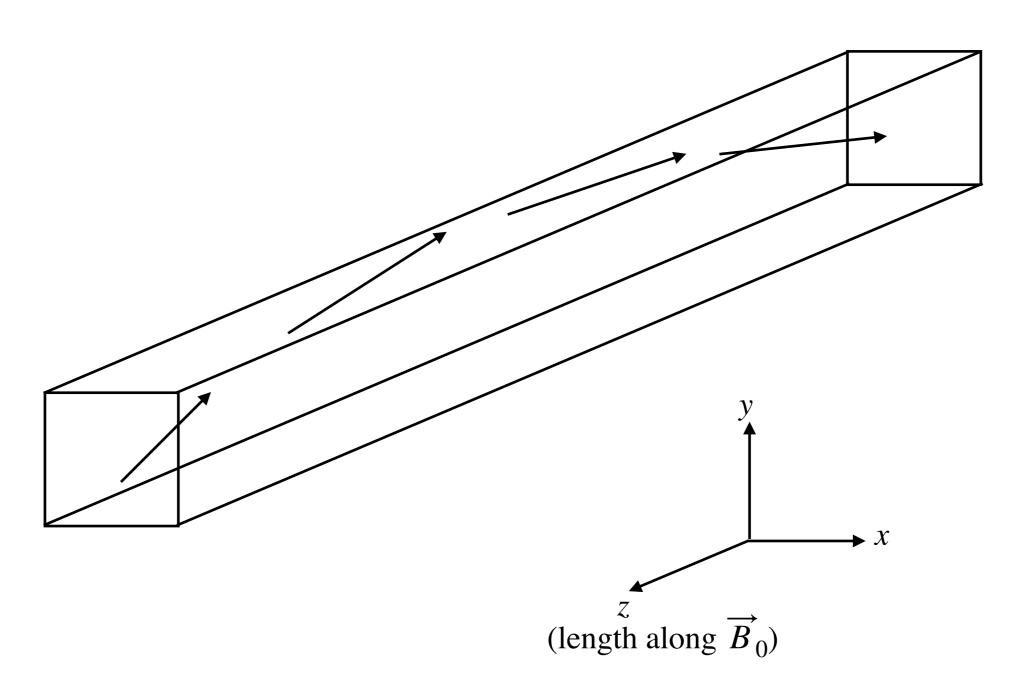




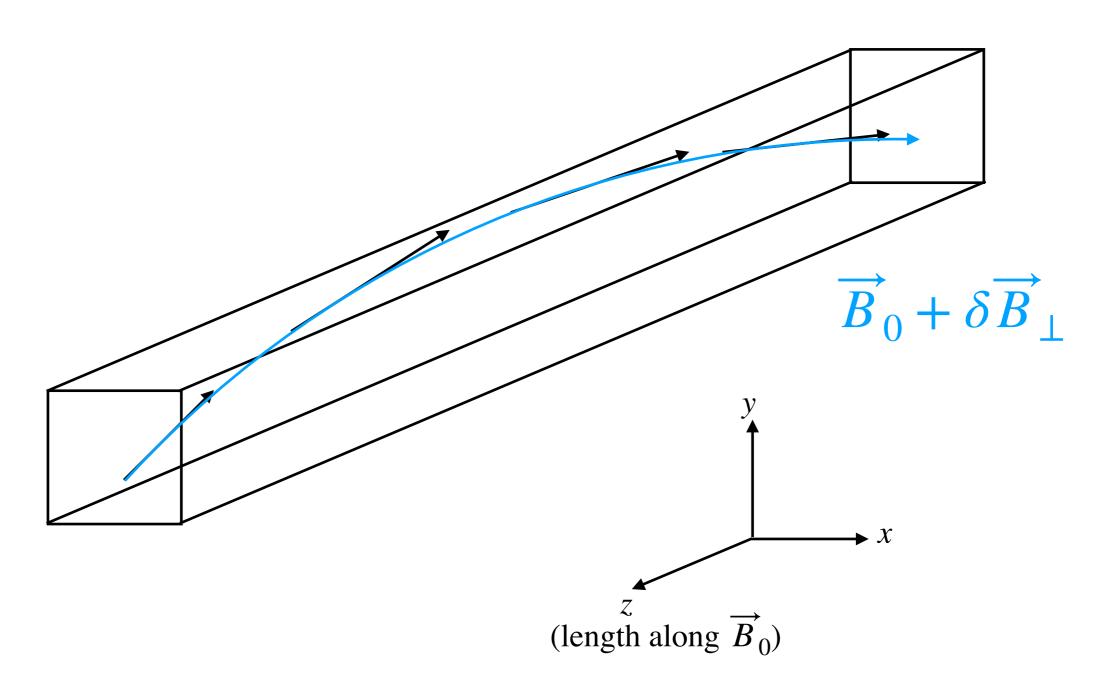




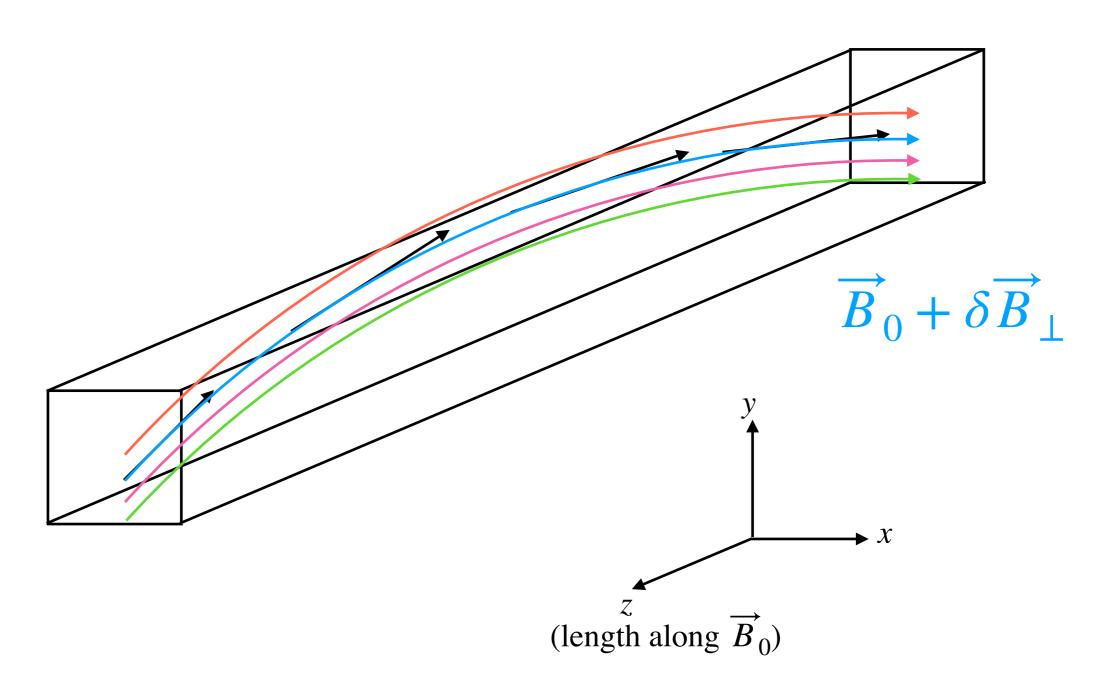




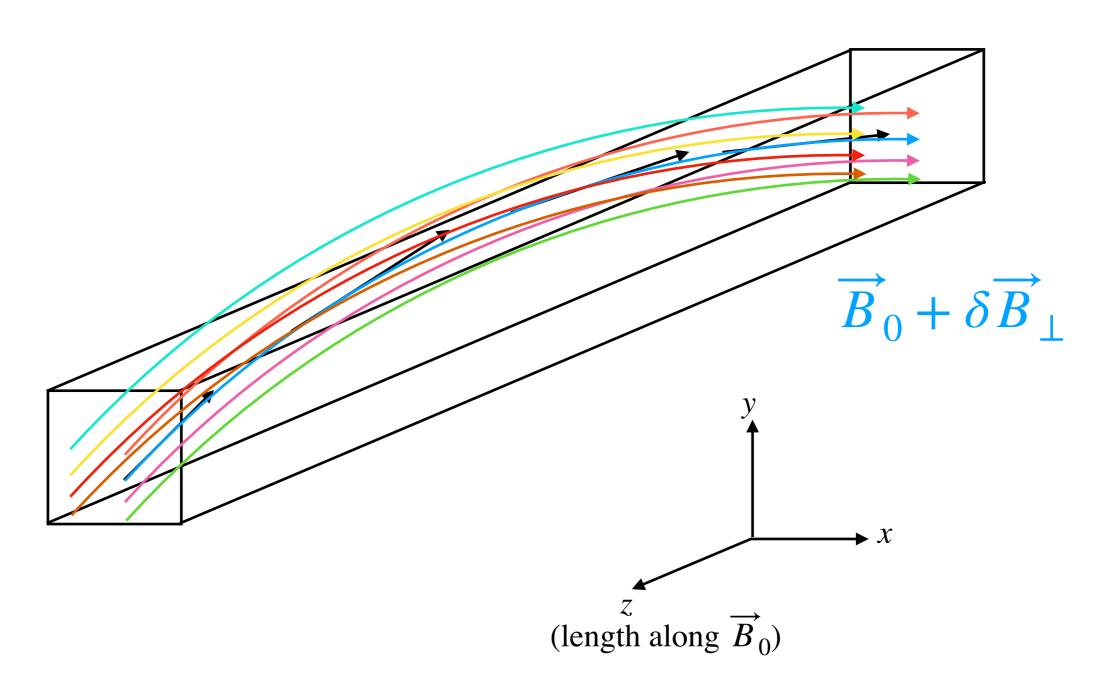




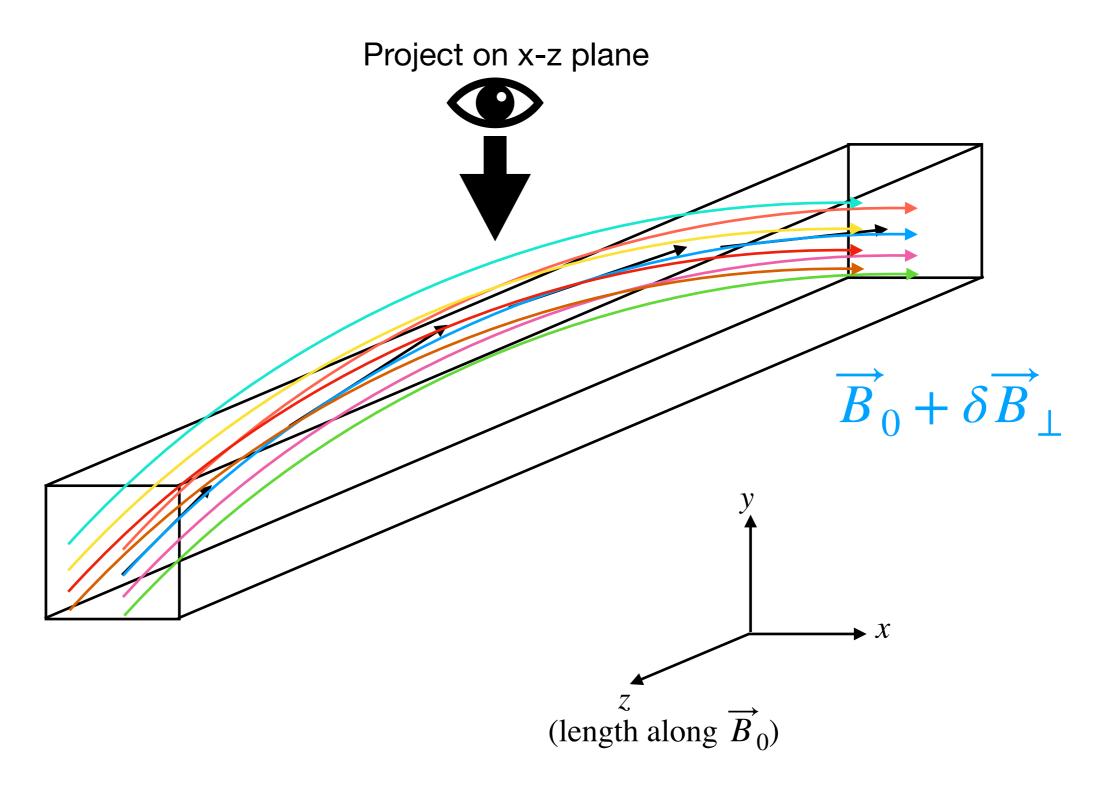




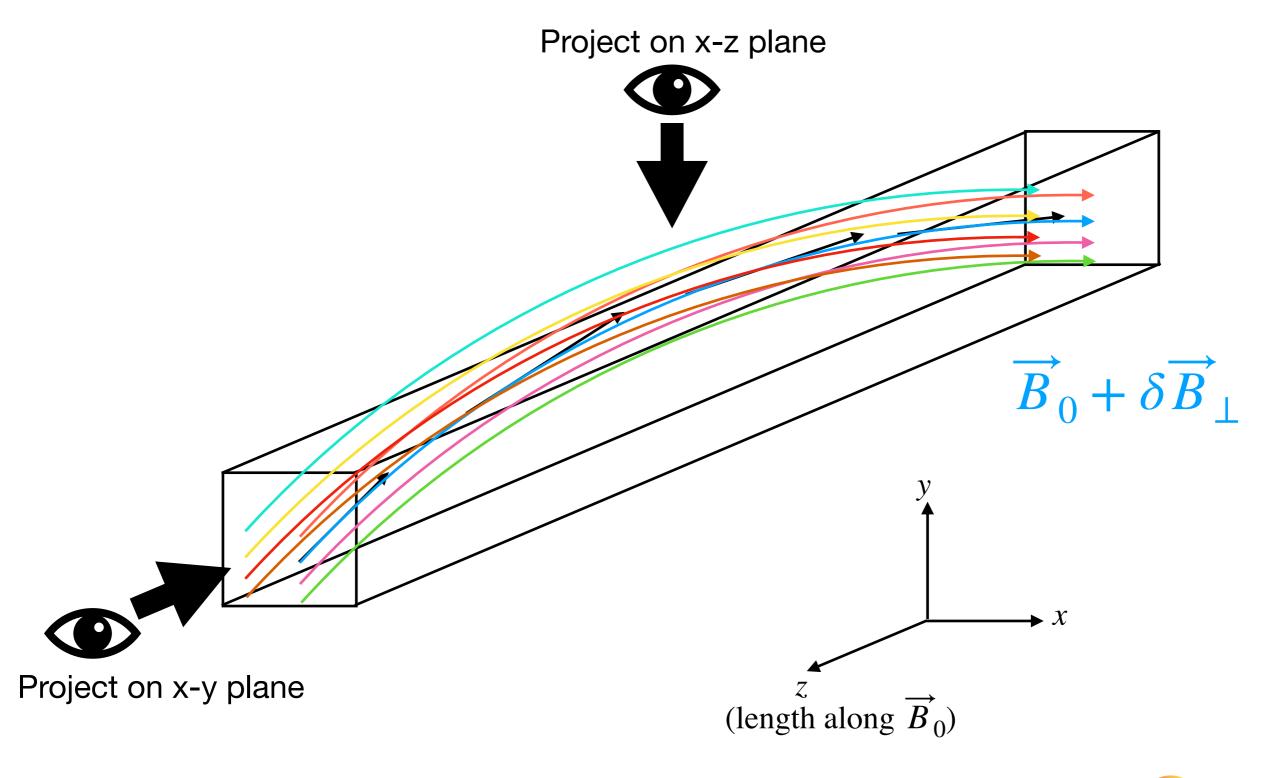




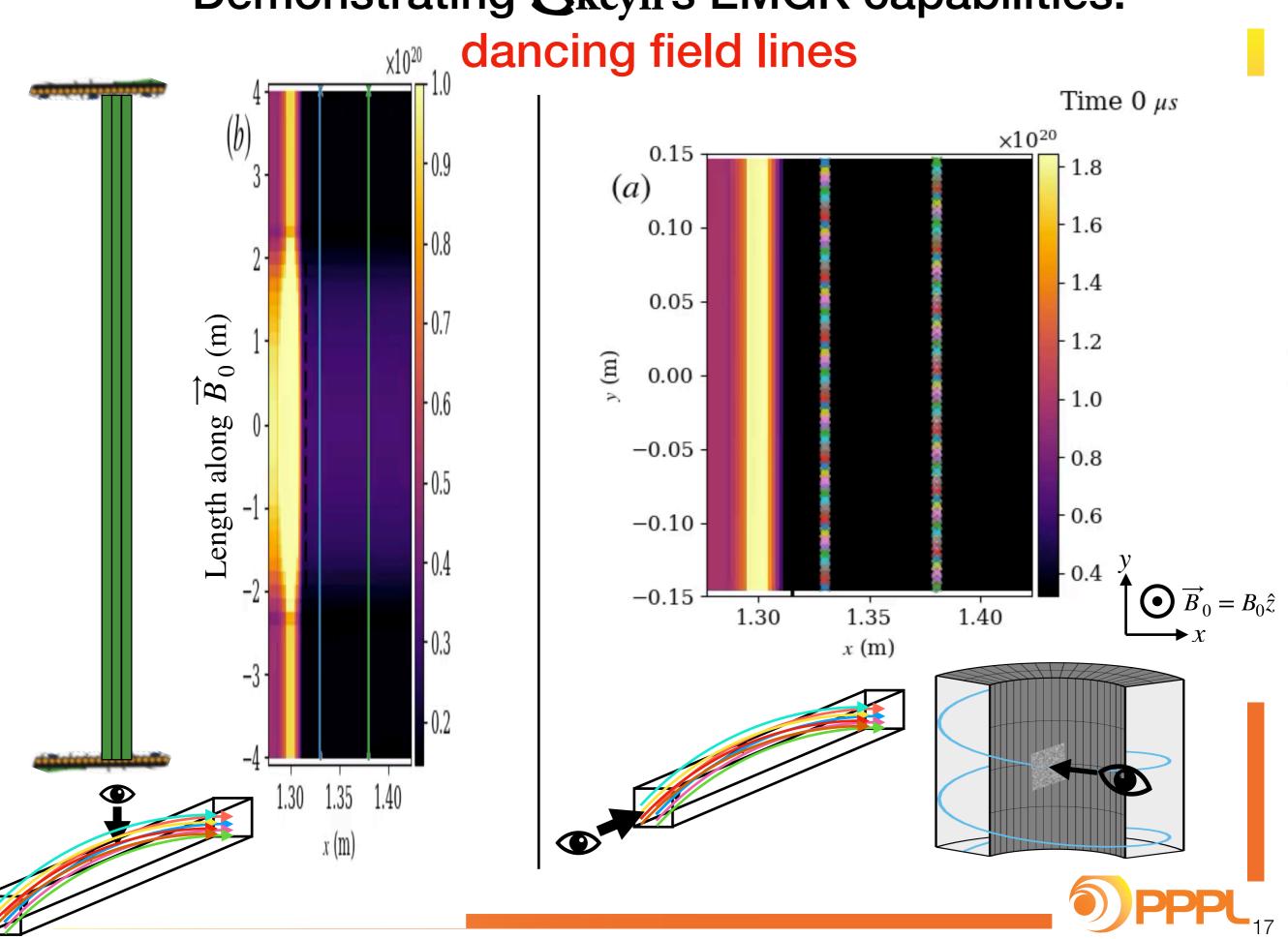


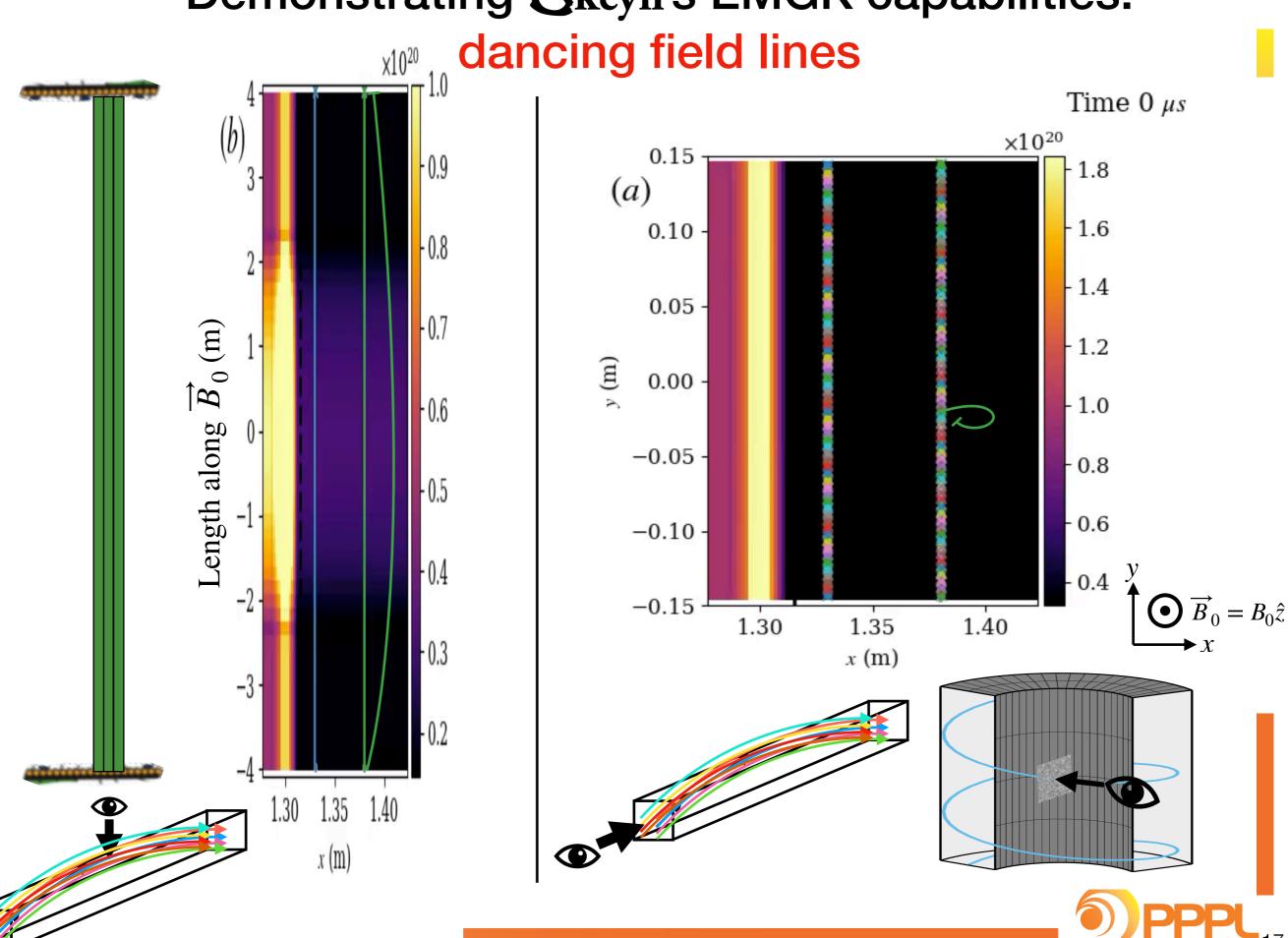


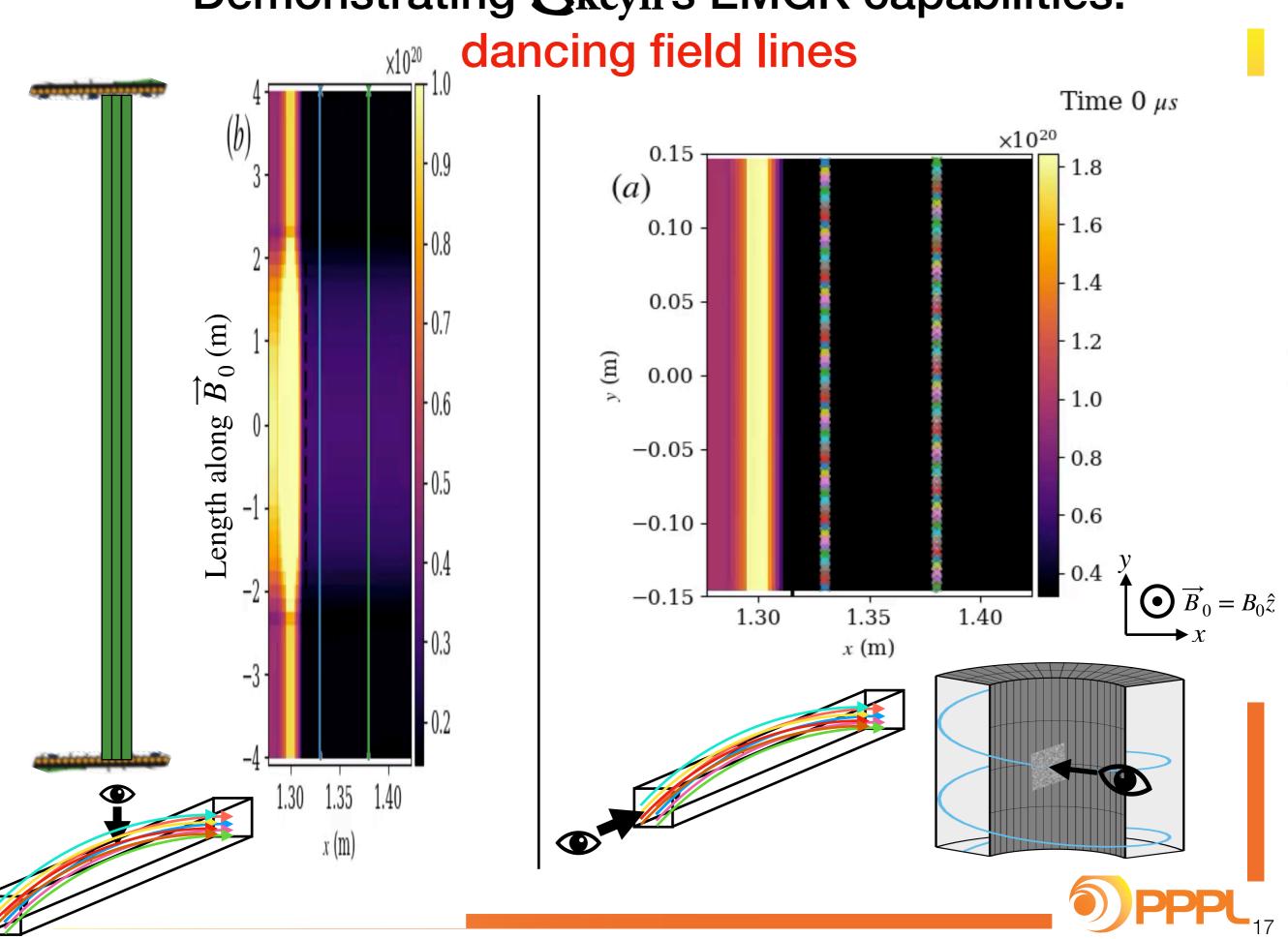


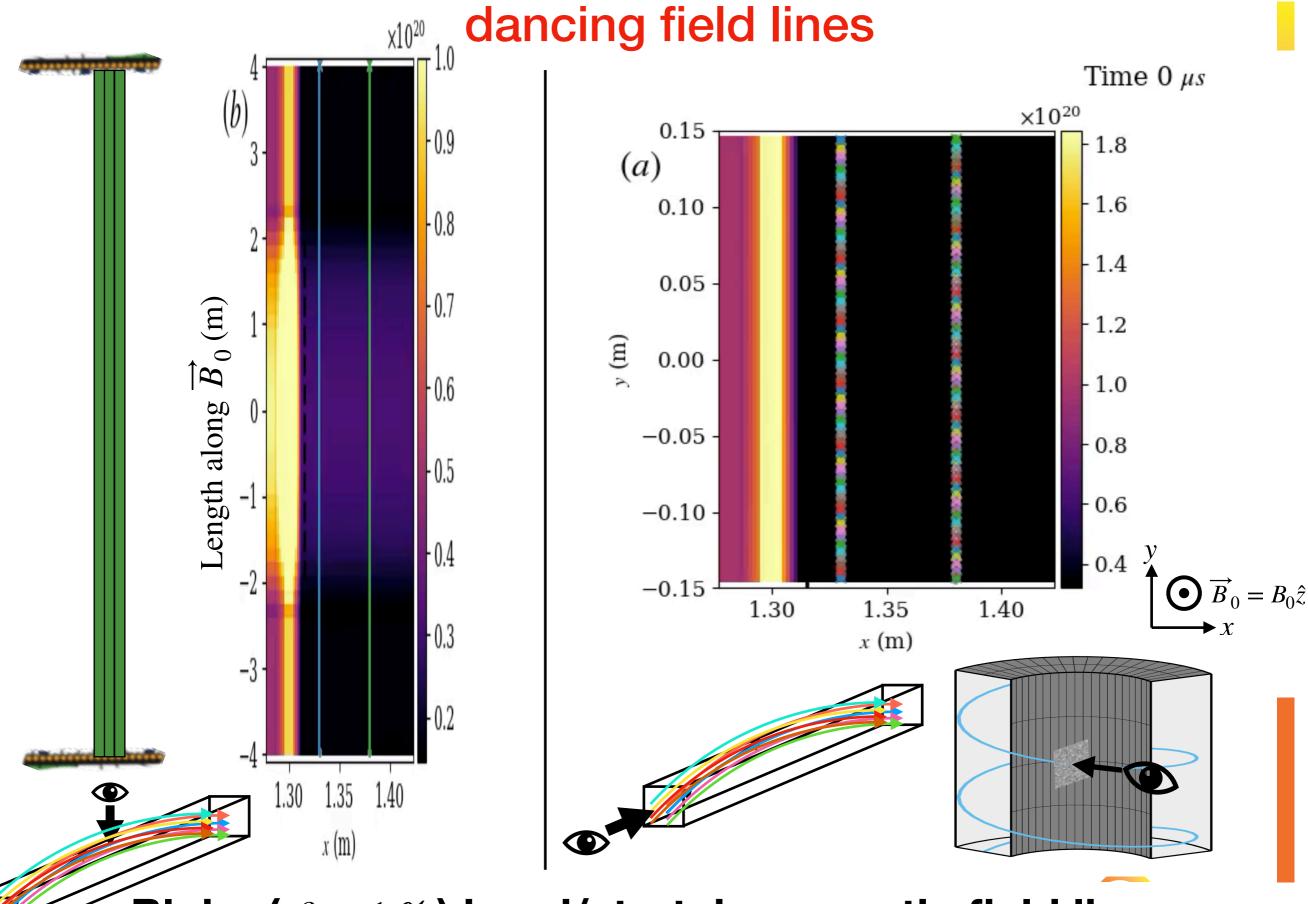












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- We've used simple helical geometry and parameters from NSTX H-mode SOL, but with  $10x n_0$  to stress-test EM effects (could happen locally in ELM?)
  - Results in magnetic fluctuations  $\delta B_{\perp}/B_0 \sim 1 \%$
  - **Ckeyll** can handle this strong magnetic turbulence robustly
  - Mandell et al, JPP 2020; Hakim et al, PoP 2020



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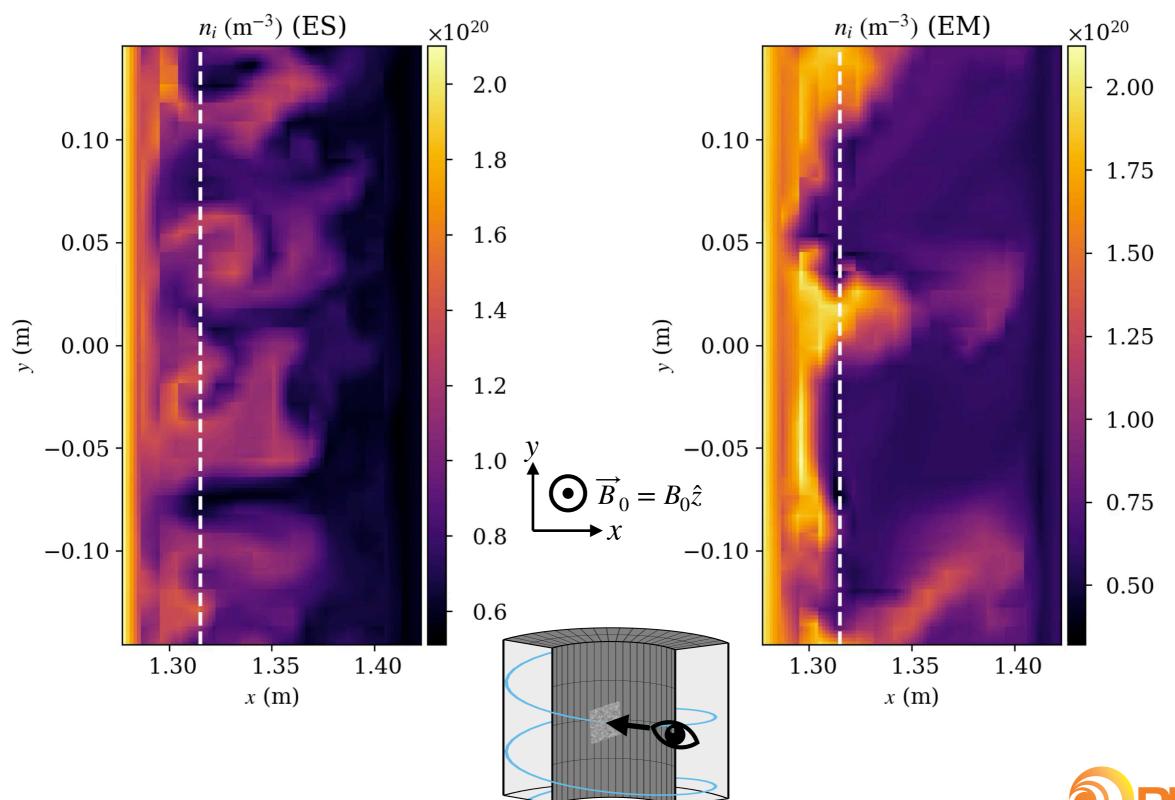
### Do EM fluctuations affect SOL turbulence dynamics?

- Now going to do side-by-side comparison of electrostatic and electromagnetic cases. Things to look for:
  - Changes in the blob structures, dynamics, frequency, etc.



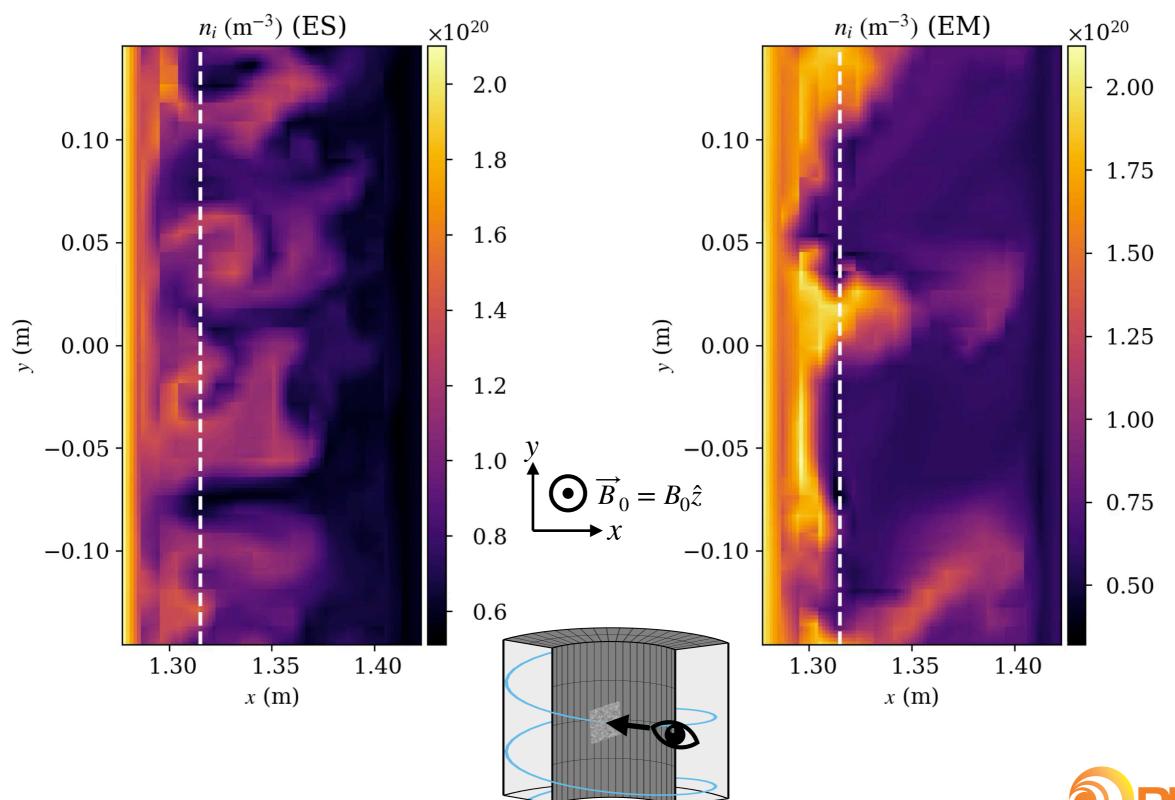
# Electrostatic/electromagnetic comparison: midplane ion density

Time 500  $\mu s$ 

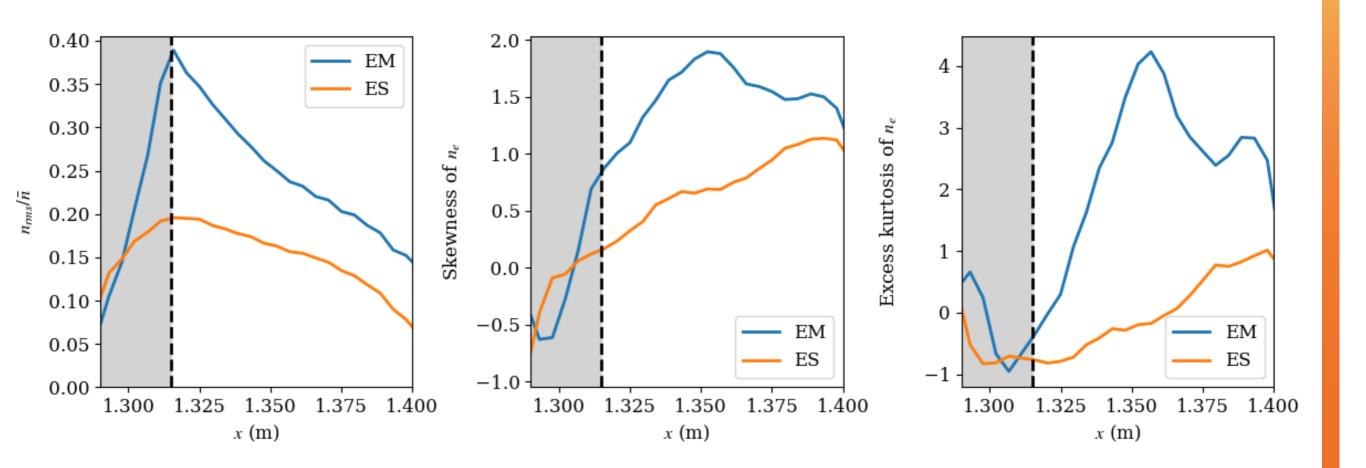


# Electrostatic/electromagnetic comparison: midplane ion density

Time 500  $\mu s$ 



# Electrostatic/electromagnetic comparison: density fluctuations statistics

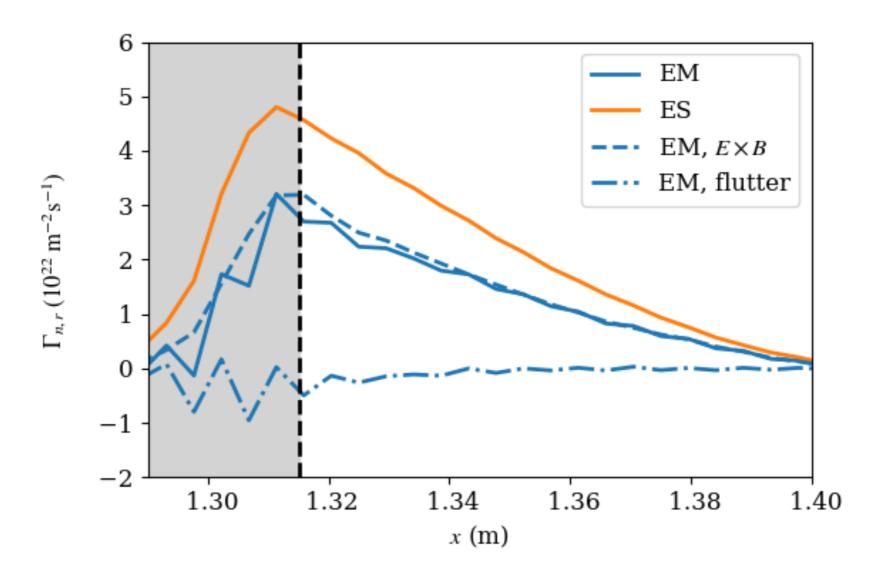


EM has larger, more intermittent density fluctuations



## Electrostatic/electromagnetic comparison: radial particle flux (near midplane)

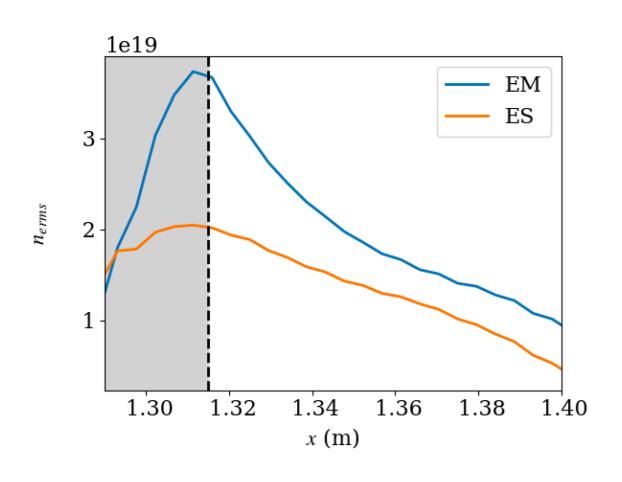
Might expect larger density fluctuations means more transport, but...

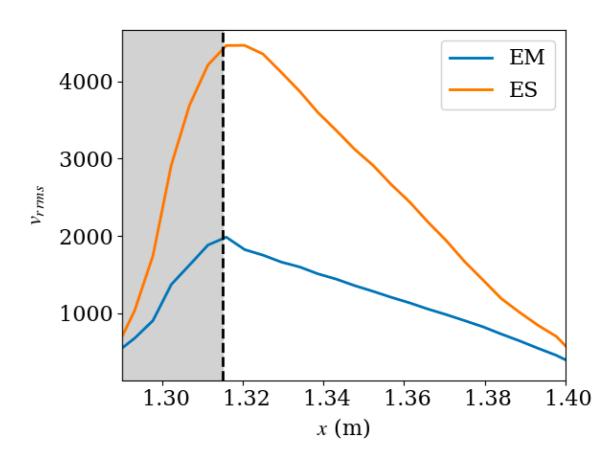


Radial particle transport reduced in EM case by ~ 40%



# Electrostatic/electromagnetic comparison: radial particle flux (near midplane)

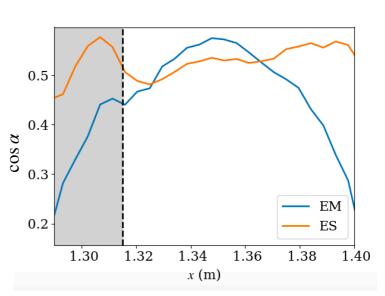




The radial  $E \times B$  particle flux is defined as

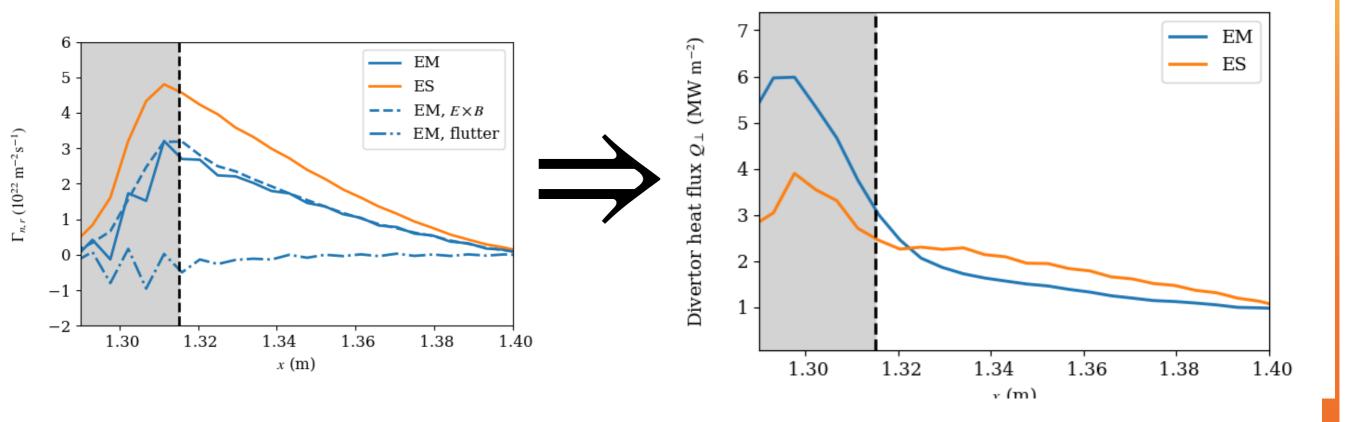
$$\Gamma_r = \langle \tilde{n}_e \tilde{v}_r \rangle = n_{e, rms} v_{r, rms} \cos \alpha$$

In this case, using  $n_{rms}$  as a surrogate diagnostic for transport is not sufficient!





# Electrostatic/electromagnetic comparison: divertor heat flux profile

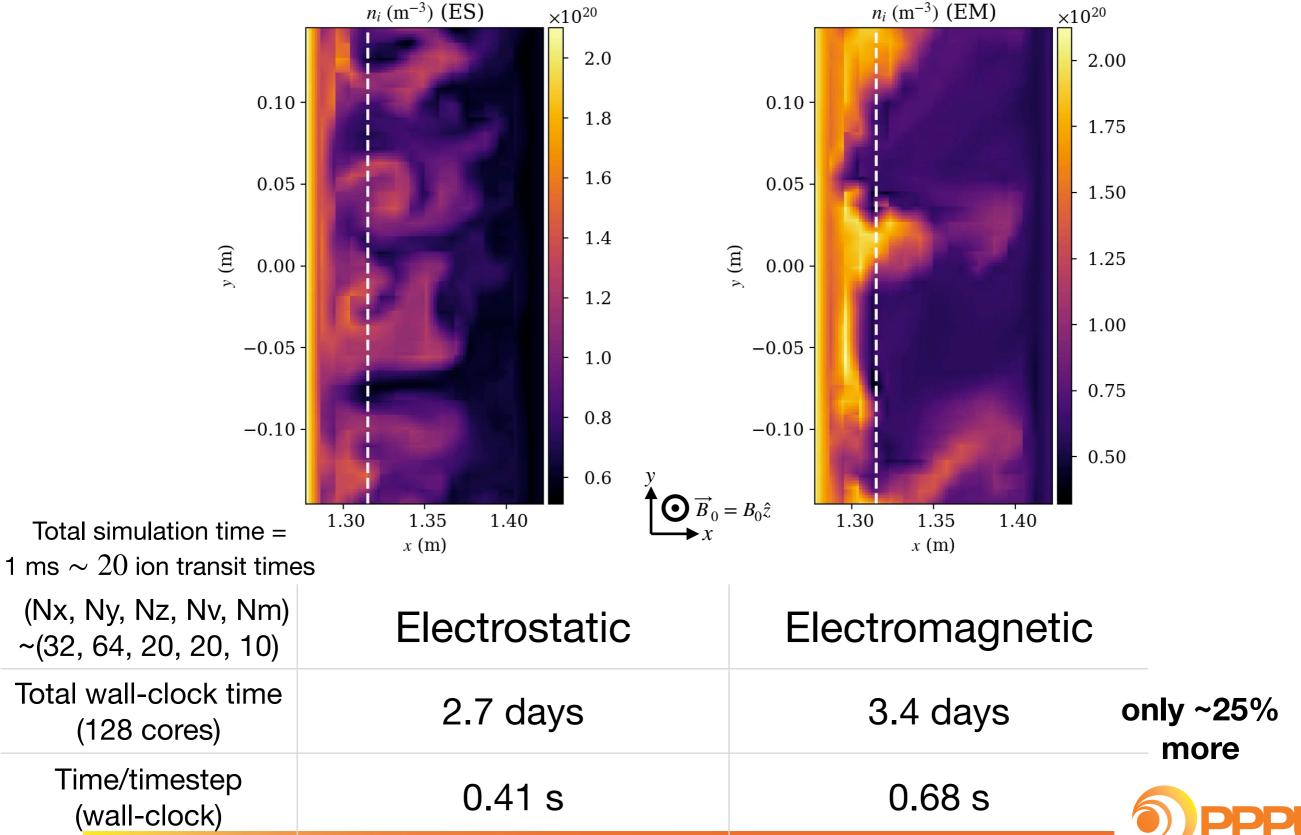


- Because EM case has less radial transport, heat flux to divertor is more peaked in EM case
- ES case over-predicts transport, over-predicts heat flux width



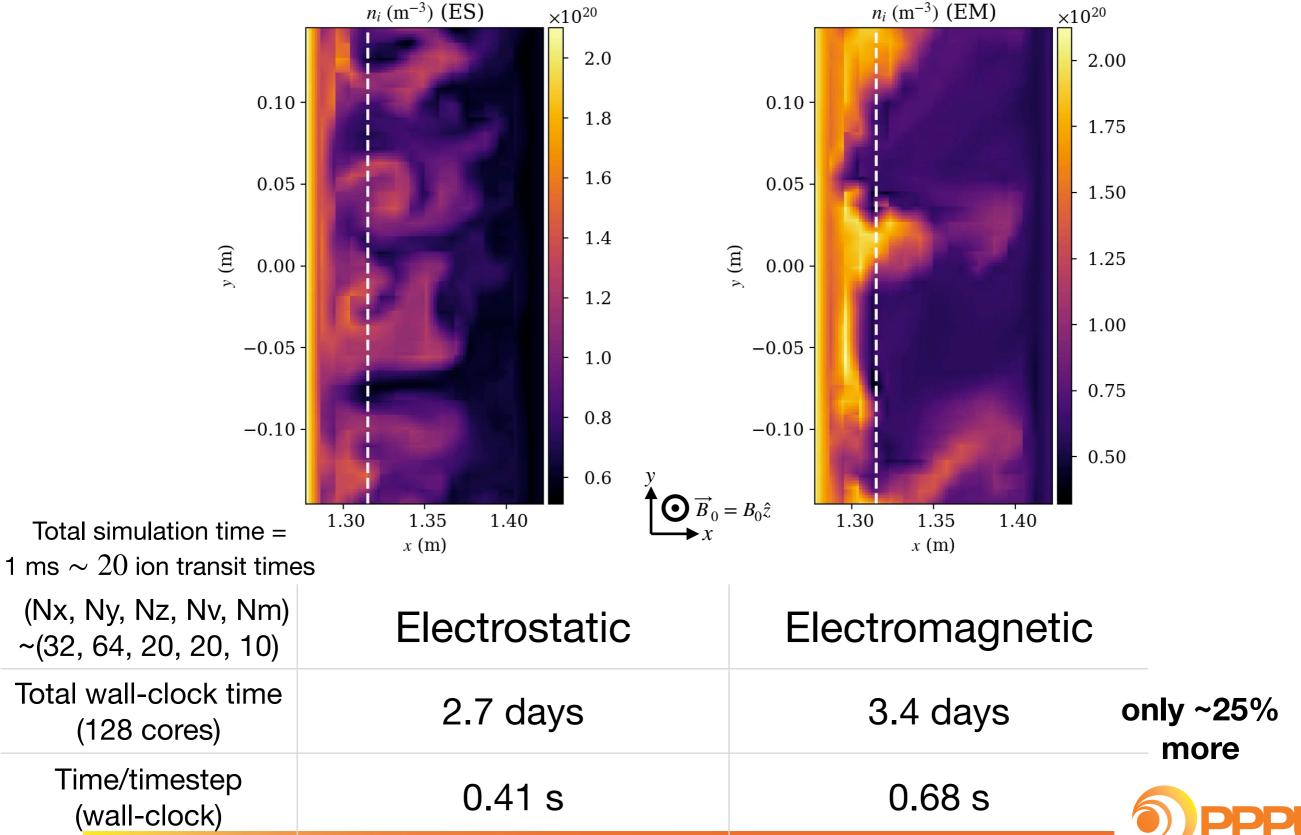
### Modest simulation cost (even for EM!)





### Modest simulation cost (even for EM!)





#### Towards more realistic SOL geometry

- We know magnetic shear can be important in SOL, especially near X point
- All Gkeyll results to date (NSTX and Helimak) have used simplified helical geometry
  - Neglected most geometrical factors, no magnetic shear
- Even in SMT configuration (e.g. Helimak) with const vertical field, there should be some magnetic shear because toroidal field  $\sim 1/R$

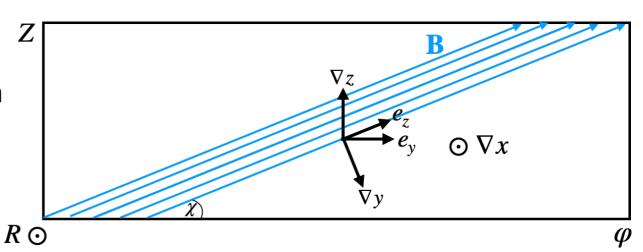
$$\mathbf{B} = \frac{B_0 R_0}{R} \hat{\boldsymbol{\varphi}} + B_v \hat{\mathbf{Z}}, \qquad q(R) = \frac{H B_{\varphi}}{2\pi R B_v} = \frac{B_0 R_0 H}{2\pi R^2 B_v}, \qquad \hat{s} = \frac{R}{q} \frac{dq}{dR} = -2$$

• Keeping helical configuration, can adjust shear by making  $B_v = B_v(R) = B_{v0}(R/x_0)^n$ , so that

$$\hat{s} = -2 - \frac{R}{B_v} \frac{dB_v}{dR} = -2 - n$$

Take field-aligned helical coordinate system with

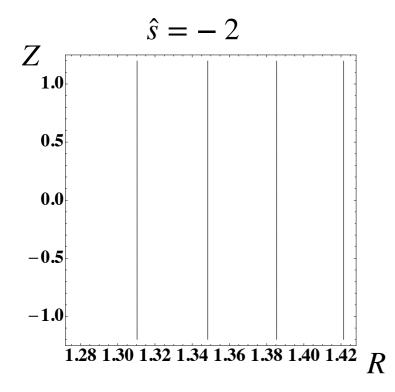
$$x = R,$$
  $z = Z,$   $y = x_0 \left( \varphi - \frac{2\pi qZ}{H} \right),$  
$$\mathbf{B} = \frac{RB_v}{x_0} \nabla x \times \nabla y$$

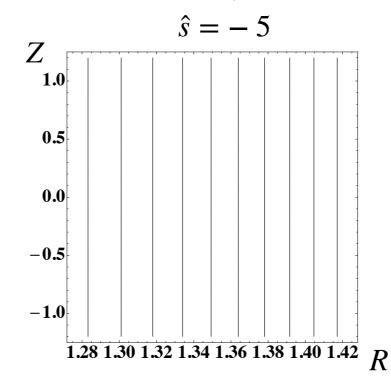


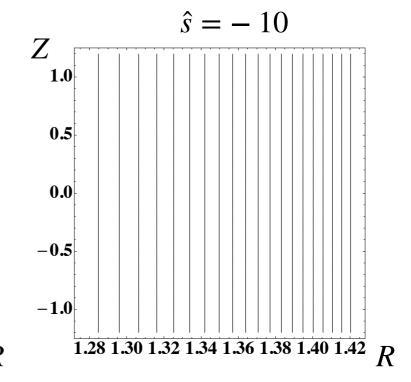


#### Helical geometry with magnetic shear

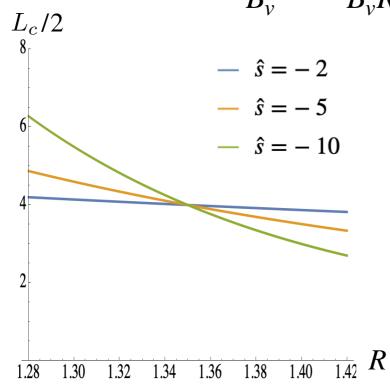
Vertical (~poloidal) flux:  $\Psi(R,Z) = R^2 B_v / 2 \sim R^{-\hat{s}}$ 

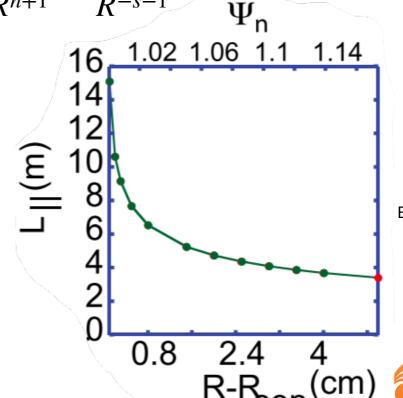






Connection length: 
$$L_c = \frac{HB}{B_v} \approx \frac{HB_0R_0}{B_vR} \sim \frac{1}{R^{n+1}} \sim \frac{1}{R^{-\hat{s}-1}}$$

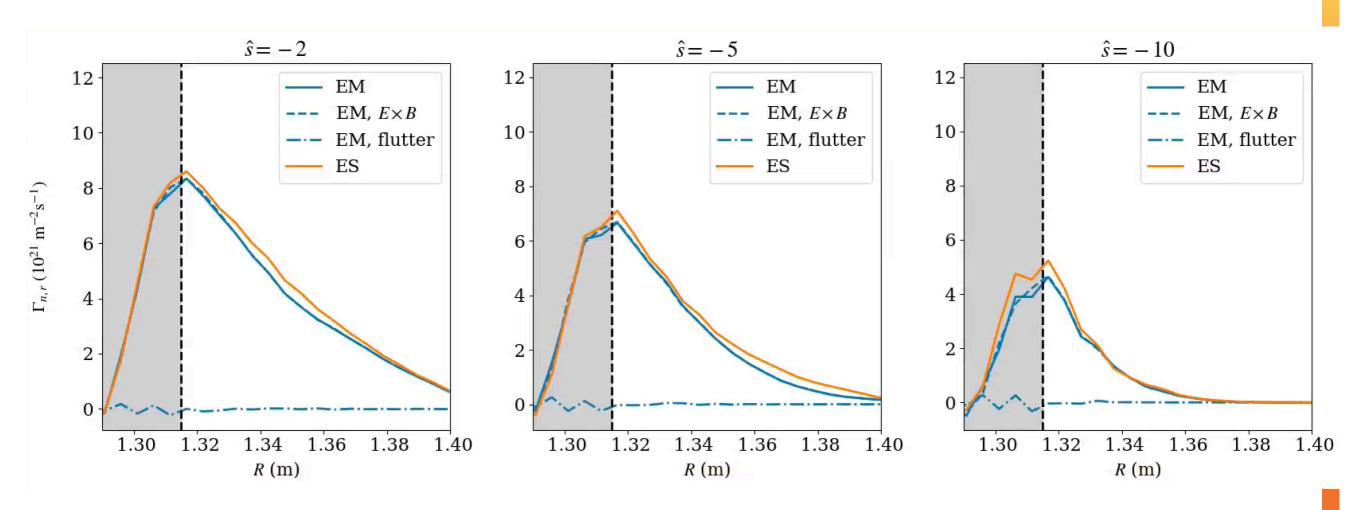




**NSTX SOL** connection length Boedo et al, PoP 2014



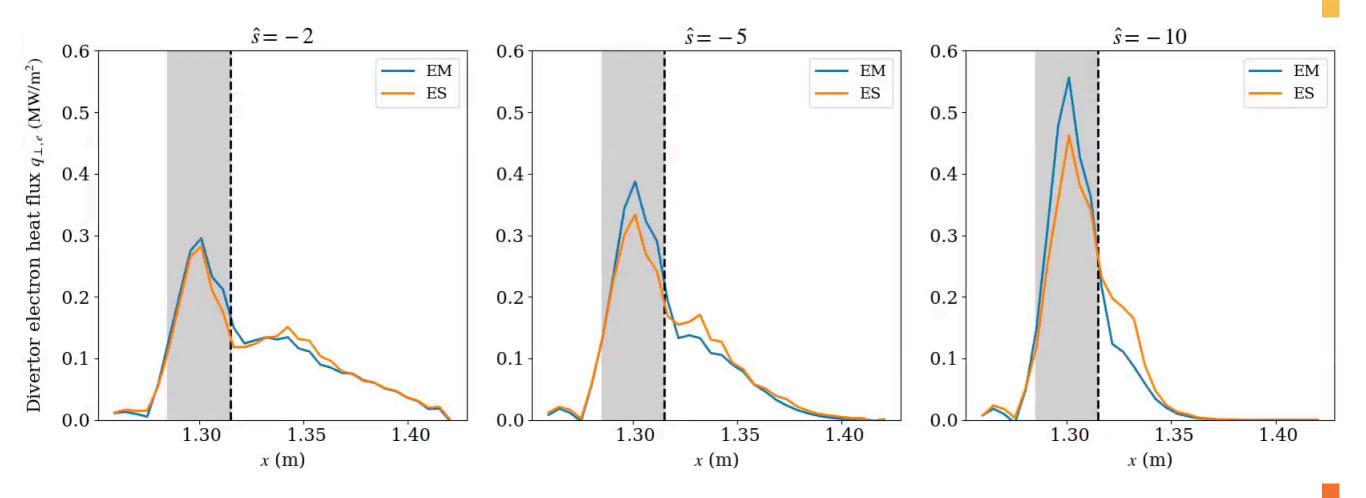
#### $\hat{s}$ scan: radial particle flux (near midplane)



- Transport decreases as  $|\hat{s}|$  increases
- EM cases again have less transport
- This is at ~ experimental  $\beta \sim 0.1\,\%$  (no more 10x)



#### $\hat{s}$ scan: divertor heat flux profiles

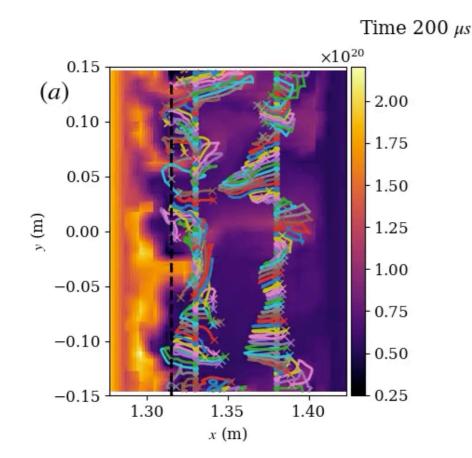


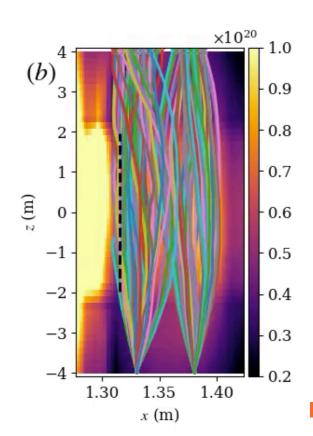
- Heat flux profile gets narrower as  $|\hat{s}|$  increases
- EM effects more important as  $|\hat{s}|$  increases



### Summary

- **Ckeyll** is being used to study SOL turbulence in tokamaks like NSTX (only handles open field lines right now)
- **Ckeyll** has produced the first nonlinear electromagnetic gyrokinetic simulations in the SOL, can handle strong magnetic turbulence with  $\delta B_{\perp}/B_0 \sim 1\,\%$
- In high  $\beta$  regime, including electromagnetic fluctuations results in larger, more intermittent fluctuations in SOL but less transport
- Moving towards more realistic SOL geometry, including magnetic shear

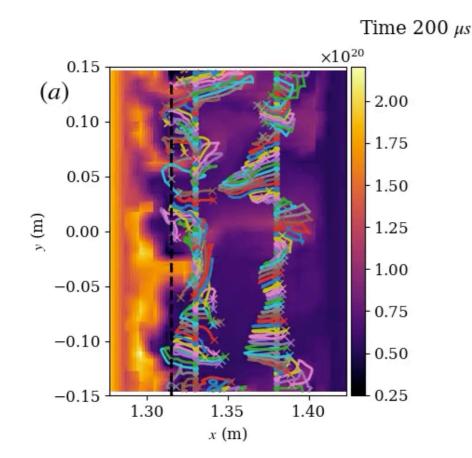


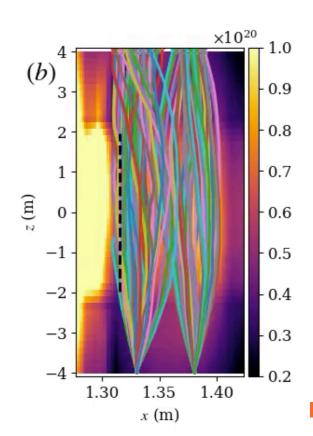




### Summary

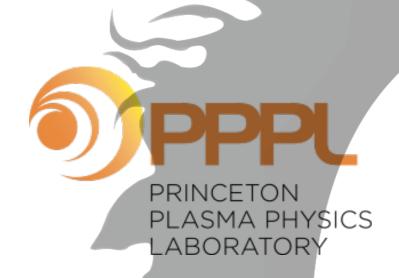
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### Acknowledgements







**6** keyll team: **Ammar Hakim Greg Hammett Tess Bernard Petr Cagas** Mana Francisquez **Jimmy Juno** Rupak Mukherjee **Liang Wang Eric Shi** 

... and others!

https://github.com/ammarhakim/gkyl/

https://gkeyll.readthedocs.io



### Gkeyll GK references

E. L. Shi, Princeton Ph.D. thesis 2017

E. L. Shi et al, J. Plasma Phys. 83 (3) (2017) 905830304

E. L. Shi et al, Phys. Plasmas 26 (1) (2019) 012307

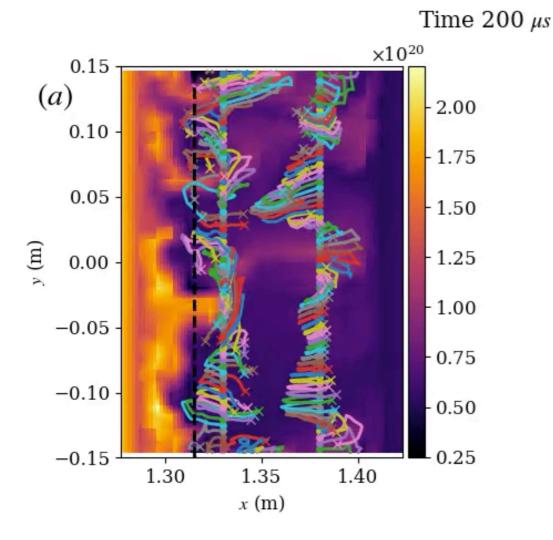
T. Bernard et al, Phys. Plasmas 26 (4) (2019) 042301

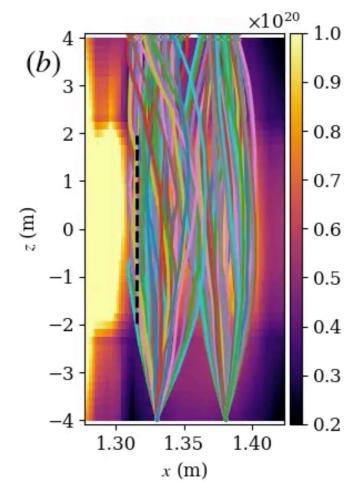
N. D. Mandall et al. I. Diagna Diagna C

A. Hakim et al, arXiv:1908.01814

N. R. Mandell et al, J. Plasma Phys. 86 (1) (2020)

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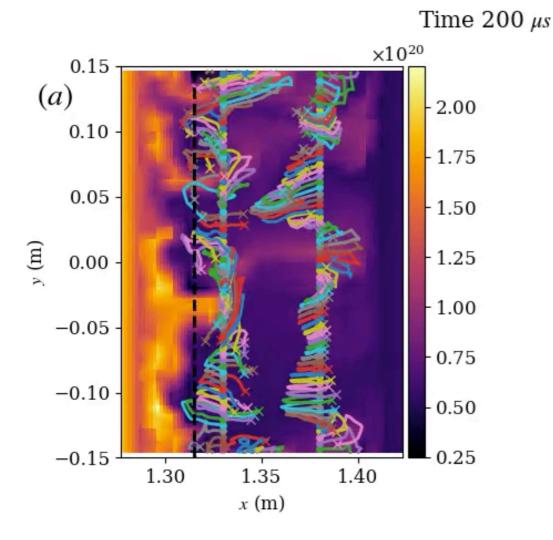
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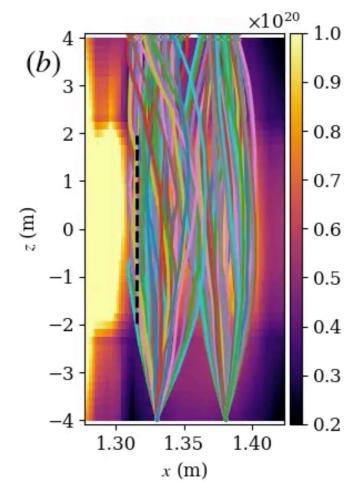
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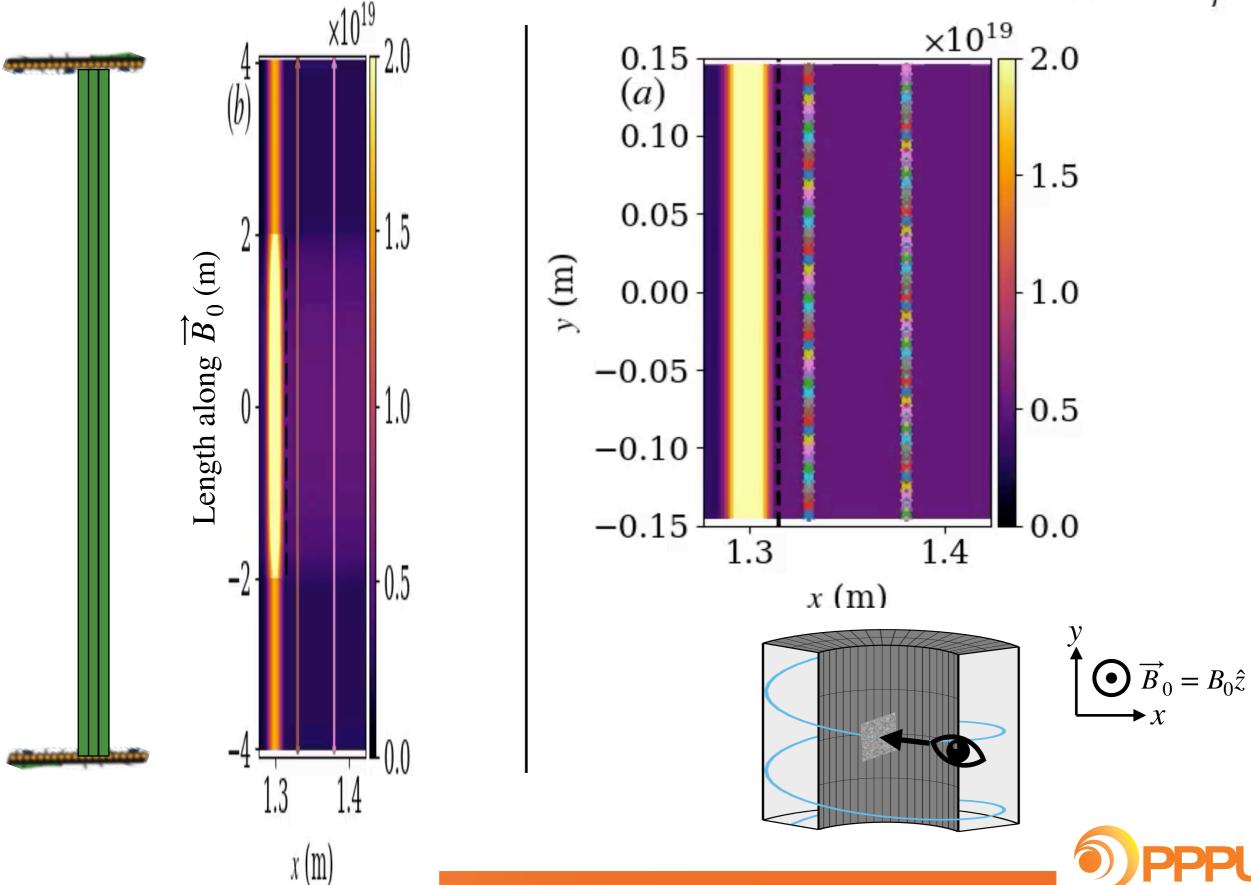




# Back-up slides

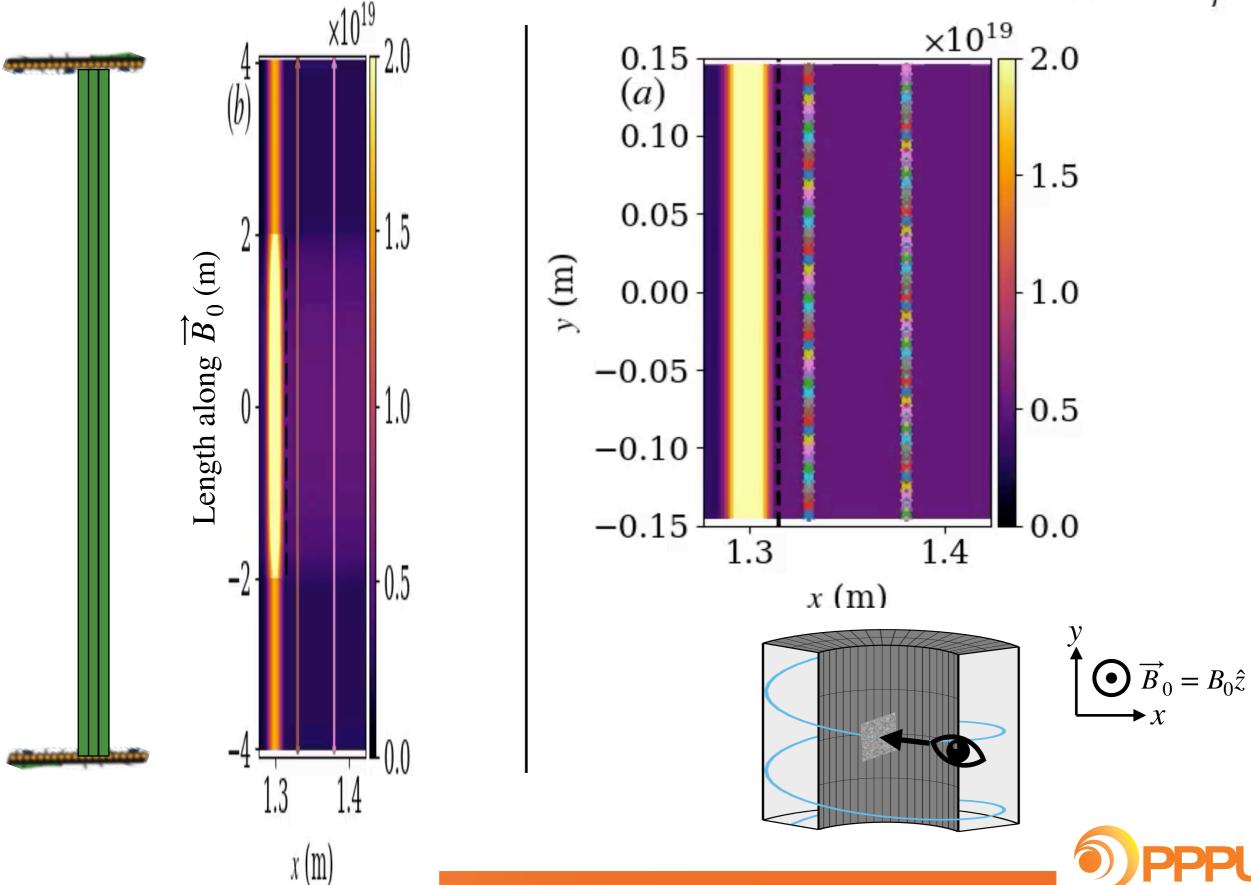
#### Dancing field lines at $\sim$ experimental $\beta$

Time 0  $\mu s$ 



#### Dancing field lines at $\sim$ experimental $\beta$

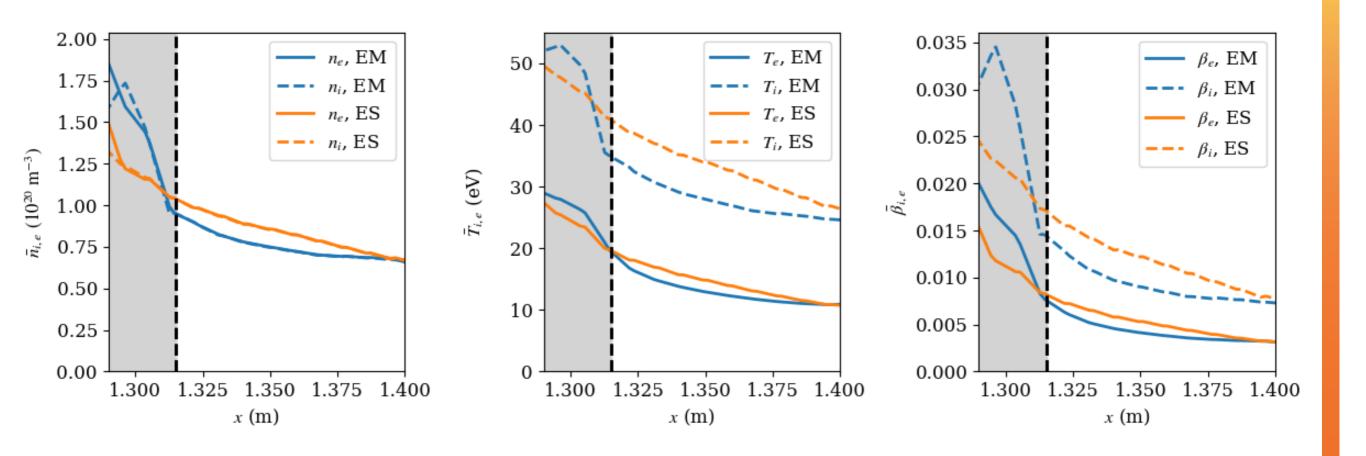
Time 0  $\mu s$ 



#### Dancing field lines at $\sim$ experimental $\beta$ Time 0 $\mu s$ $\times 10^{19}$ 0.15 (a) 0.10 -1.5 0.05 -Length along $\overrightarrow{B}_0$ (m) y (m) 0.00 -1.0 -0.05-0.5-0.10-0.151.4 -0.5x (m) $\bigodot \overrightarrow{B}_0 = B_0 \hat{z}$

But increasing connection length, adding magnetic shear can also increase magnetic fluctuations and affect transport

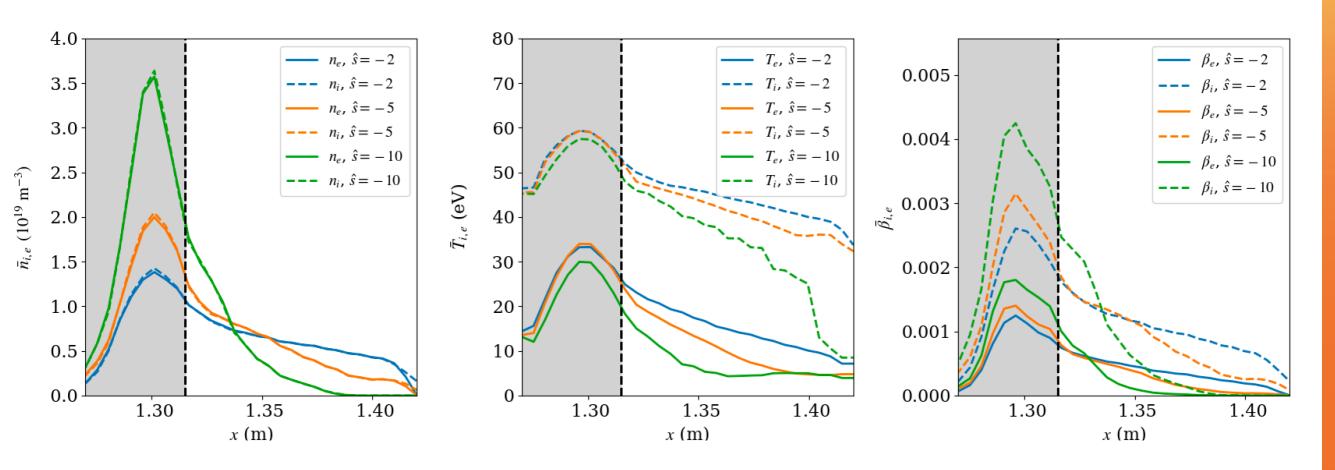
## Electrostatic/electromagnetic comparison: midplane profiles



- $\beta_e \sim 0.5 \,\%$  ,  $\beta_i \sim 1 \,\%$  in SOL region (  $\sim 10 \,\times$  NSTX SOL )
- Profiles are steeper in source region, shallower in SOL region in EM case



### $\hat{s}$ scan: midplane profiles (EM)

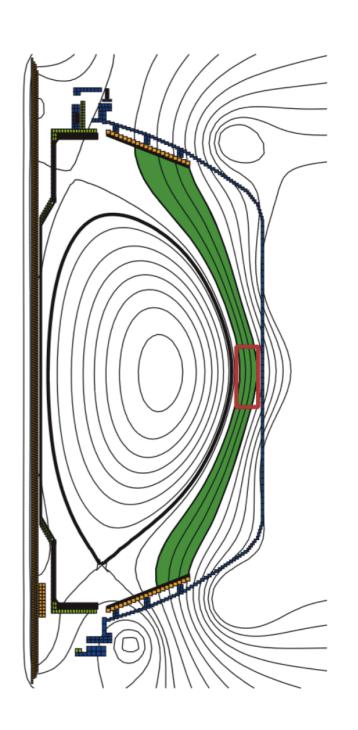


- $\beta \sim 0.1\%$  (~ NSTX SOL)
- Profiles drop off more quickly as  $|\hat{s}|$  increases
- Previous simulations with simplified geometry similar to  $\hat{s} = -2$  case



### **Current/Future Work**

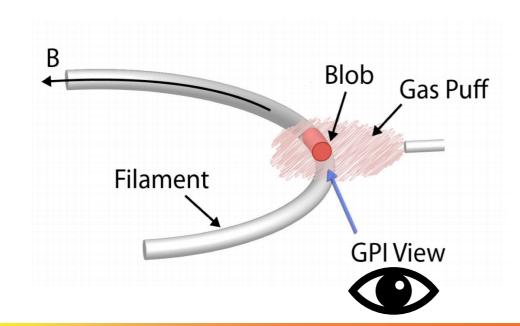
- Generalizing the magnetic geometry to include magnetic shear, non-constant curvature, closed field line regions, X-point
  - Non-orthogonal field-aligned coordinate system with magnetic shear now implemented
  - X-point is a singularity in these coordinates, challenging!
- More studies of EM effects on blobs/ELMs
  - Comparisons with magnetic fluctuation measurements in experiments?

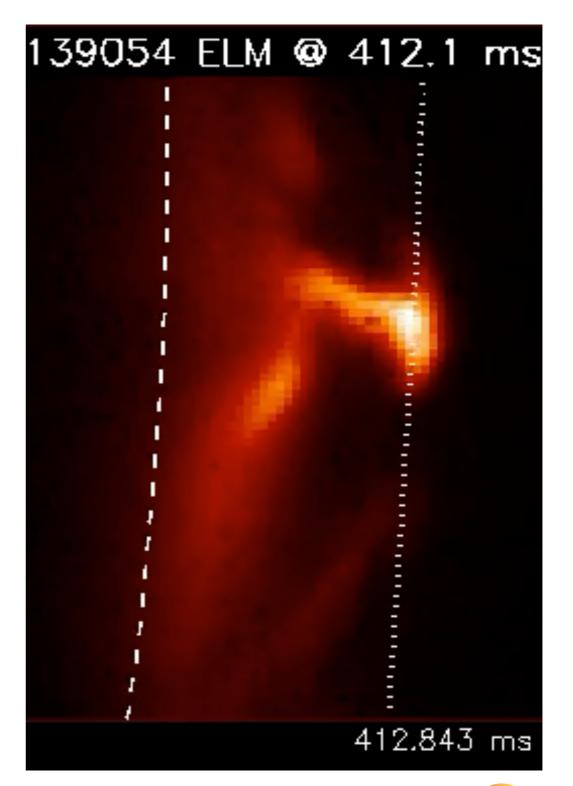




### Imaging the SOL with GPI

- GPI = Gas-puff imaging diagnostic (S. Zweben)
- Real-time turbulence movies in NSTX SOL
- Data taken using fast camera (400,000 fr/s)
- $\mathrm{D}\alpha$  intensity proportional to some combination of n and T

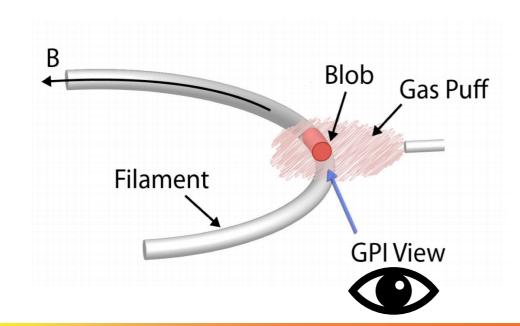


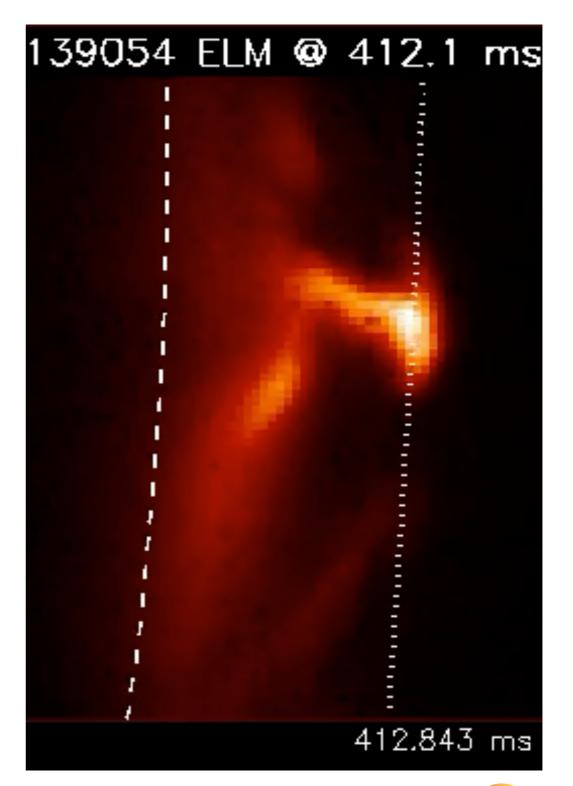




### Imaging the SOL with GPI

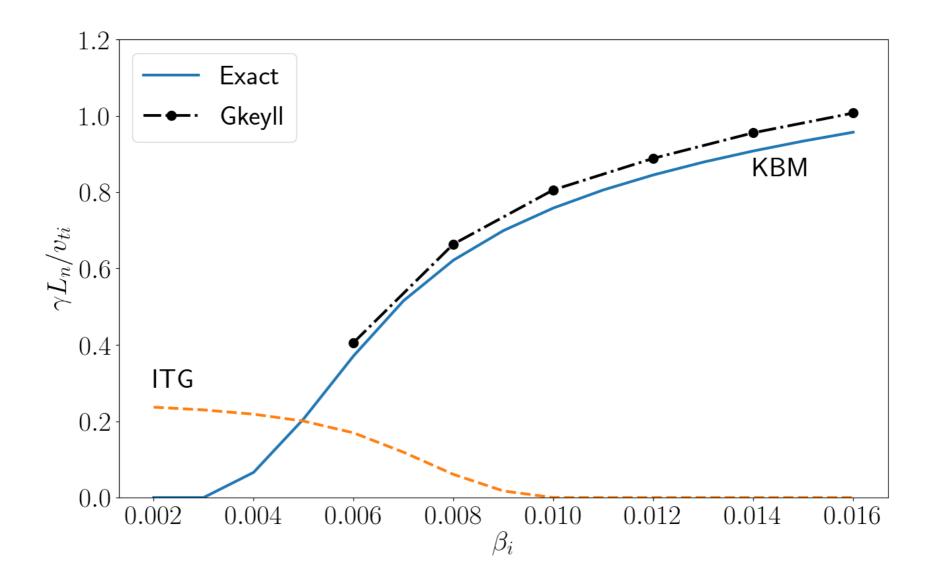
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- Real-time turbulence movies in NSTX SOL
- Data taken using fast camera (400,000 fr/s)
- $\mathrm{D}\alpha$  intensity proportional to some combination of n and T







#### Linear benchmark: KBM instability (local limit)



$$k_{\perp}\rho_{s}=0.5,\ k_{\parallel}L_{n}=0.1,\ R/L_{n}=5,\ R/L_{Ti}=12.5,\ R/L_{Te}=10,\ \tau=1$$

